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October 10, 1973

NIC #19461
Packet Radio Temporary Note #75

PERFORMANCE OF CARRIER SENSE WITH HIDDEN TERMINALS

1. INTRODUCTION

1.1 Review

Several versions of carrier-sense access models have been introduced and analyzed so far. For easy reference, we identify these versions by the following:

a. The 1-Persistent Carrier Sense (or auto slot) was presented and analyzed in Packet Radio Temporary Note #24 [K1]. In this case, a terminal with a packet ready for transmission senses the channel and operates according to the following protocol:

- if the channel is idle, it transmits the packet with probability 1.
- if the channel is sensed busy, it waits until the channel gets idle (persisting on transmitting) and then transmits the packet (with probability 1).

b. The p-Persistent Carrier Sense (or carrier sense with initial random transmission delay) was introduced and studied in Packet Radio Temporary Note #37 [K2]. This version of carrier sense differs from the previous one mainly by the fact that when the channel is sensed idle or gets idle after having been busy, the terminal transmits the packet with a probability p and delays its transmission by τ sec. with a probability $1 - p$.

c. The Non-Persistent Carrier Sense (or carrier sense with randomized resensing delay) was studied in Packet Radio Temporary Note #51* [F1]. In this case, when the terminal senses the channel idle it transmits the packet, and when it senses the channel busy, it reschedules the transmission of the packet for some random time later (thus delaying it with probability 1).

Previous results were based on the assumption that each terminal could hear all traffic supported by the channel. In more realistic environments, the problem of interference caused by some terminals "hidden" from the transmitter(s) is quite clear.

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*We give in Appendix A a brief discussion of the analysis presented in SRI's Packet Radio Temporary Note #51 and correct their expression for throughput. We also compare the throughput-delay tradeoff for the three versions described above.

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1.2 Purpose

The purpose of the study presented in this note is to gain some insight into the effect that Hidden Terminals can have on the performance of carrier sense access modes.

For the sake of simplicity (in analysis), we restrict ourselves at this point to both the 1-Persistent and the Non-Persistent Carrier Sense access modes. Whenever possible, an effort is made to provide general analytic models, and their evaluation is compared to simulation results.

1.3 Results

Several particular configurations are considered as examples. Specific observations concerning each configuration are discussed in the text. Our main result is that the performance of carrier sense (throughput-wise) is badly degraded by the existence of hidden terminals. This shows the great importance of considering the hidden terminal problem and seeking a solution to it. The "busy tone" access mode [F1] is a good candidate which we plan on investigating.

2. The Environment Under Consideration. Representation of Terminal Configurations with Hidden Elements

The environment we are focusing on is still the case of a large population of small and independent users, all within range of, and communicating with a station in LOS of all terminals (the entire population). Any configuration with hidden terminals can generally be represented as follows:

The population is partitioned into several (say N) groups. Let G denote the set of groups numbered 1 to N. All terminals within a group can hear each other. We define a graph with N nodes and make a one-to-one correspondence between the nodes of the graph and the N groups defined above. A link between two nodes i and j represents the fact that group i and group j hear each other.

The above notions can be represented equivalently by defining a matrix with N rows and N columns. The value of an element a_{ij} of the matrix is defined by:

$$a_{ij} = \begin{cases} 0 & \text{if group } i \text{ does not hear group } j \\ 1 & \text{if group } i \text{ hears group } j \end{cases}$$

The matrix is of course symmetric and all elements on the main diagonal are 1.

Define the function h: $G \rightarrow 2^G$, where 2^G represents the set of all subsets of G, such that

$j \in h(i)$ if and only if $a_{ij} = 1$

i.e., $h(i)$ is the set of groups that group i can hear.

From an analytical point of view, one can distinguish two cases:

1. The case of independent groups: no links in the graph, i.e., all elements of the matrix are such that

$$a_{ij} = 0 \qquad i \neq j$$

$$a_{ii} = 1$$

2. The case of dependent groups: the general case where the nodes in the graph can be linked in any arbitrary fashion.

3. 1-Persistent Carrier Sense

Let S_i and G_i $(i = 1,2, \ldots, N)$ be respectively the normalized collective input rate and traffic rate of group i.

Assumption: All assumptions made in previous notes on the independence of arrivals and the exponential interarrival distribution for both new and retransmitted packets still hold true. Similarly, the propagation delay τ is identical for all terminals.*

3.1 Independent Group Case

Theorem 1.† Under the assumptions stated above, and in the case of independent groups, the probability of success of a packet from group i is given by:

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*In this study the assumption concerning the propagation delay becomes rather a strong one since we are concerned, in many instances, with geographical groupings. More accurate models involving non-identical propagation delays between different pairs of devices should be given more consideration. However, for a given configuration, by considering a constant propagation delay equal to the maximum encountered, our results provide lower bounds on throughput or performance. Further, these lower bounds are expected to be fairly close to exact results as long as this maximum is less than or equal to 1% of the transmission time of a packet.

†See Appendix B for proof of this theorem.

$$P_{S_i} = \frac{S_i}{G_i} = \frac{G_i + aG_i + e^{-aG_i}}{(1 + aG_i)e^{-G_i}} \prod_{j=1}^{N} \frac{(1 + aG_j)^2 e^{-G_j(2+a)}}{G_j(1 + 2a) - (1 - e^{-aG_j}) + (1 + aG_j)e^{-G_j(1+a)}} \qquad (1)$$

For a configuration of N independent groups, we may write a system of N equations relating the set $\{S_i\}$ of input rates to the set $\{G_i\}$ of traffic rates. Two questions are of interest:

Q1. Is a given set $\{S_1, S_2, \ldots S_N\}$ feasible?*

Q2. What is the relative performance of different groups?

Since the expressions given above are rather complicated, a simple characterization of the feasible set has not been found. However, numerical procedures (a less elegant but effective approach) can be used in answering Q1 and Q2. To do so we write the system of N equations in the form

$$G_i = S_i / P_{S_i}$$

and solve (1) iteratively starting with the initial values

$$G_i = S_i \qquad \forall\, i$$

The convergence of the iterative procedure determines the feasibility of the set $\{S_i\}$, and the final values G_i/S_i give a measure of the average number of transmissions per packet for each group. We refer the reader to Section 3.3 for some applications of this theorem.

3.2 Dependent Group Case

Dependence between groups renders the determination of the sequence of idle and busy periods rather difficult. No mathematical model is yet available for this case. Simulation tools have been used. An example can be found in Section 3.3, paragraph a(ii).

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*A set of input rates $\{S_i\}$ is said to be infeasible if it saturates the channel.

$$S_i' = \frac{G_i}{(1 + aG_i)e^{-G_i}} + \frac{G_i + aG_i^2 + e^{-aG_i^2}}{\prod_{j=1}^{N} G_j(1 + 2a) - (1 - e^{-G_j^2}) + (1 + aG_j)e^{-G_j(1+a)}} \prod_{j=1}^{N} (1 + aG_j)e^{-G_j(2+a)} \tag{1}$$

3.3 Some Examples

a. Symmetric Configurations

Consider the situation in which a fraction α of hidden terminals (in terms of input rates) is constant for all terminals in the population. There are many instances which fall within this category. Below we consider three such examples:

(i) The population is partitioned into N groups of equal size, mutually isolated. This instance falls within the case of independent groups. It corresponds to values of $\alpha \geq .5$, namely

$$\alpha = \frac{N-1}{N} \qquad (N > 1)$$

In this case, if S denotes the total input rate, the input rate of group i is $S_i = s = S/N$ $(i = 1, \ldots, N)$. Because of symmetry, G_i is a constant which we denote by g; $G = Ng$ is the aggregate traffic rate. Under such conditions, the application of theorem 1 gives

$$\frac{S}{G} = \frac{s}{g} = \left(\frac{g + ag + e^{-ag}}{1 + ag}\right) e^{g} \left[\frac{(1 + ag)^2 e^{-g(2+a)}}{g(1 - 2a) - (1 - e^{-ag}) + (1 + ag)e^{-g(1+a)}}\right]^N \qquad (2)$$

It is easy to see that if $N \to \infty$ such that $s \to 0$, $Ns = S$ and $g \to 0$ such that $Ng = G$, then

$$\frac{S}{G} \to e^{-2G} \qquad \text{(Aloha access mode)}$$

The maximum throughput as a function of N is plotted in Fig. 1. The point $N = 1$ $(\alpha = 0)$ corresponds to 1-persistent carrier sense without hidden terminals. Note that the maximum throughput experiences a sharp decrease between the point $N = 1$ $(\alpha = 0)$ and $N = 2$ $(\alpha = 0.5)$ (as soon as N is larger than 1, slotted Aloha performs better than carrier sense (throughput-wise)) and that it is rather insensitive for $N > 2$. In the following example, we investigate the case $0 < \alpha < .5$.

(ii) The population is divided into N groups (N even) of equal size, such that for each terminal all but 1 group are within sight. Such a geographical configuration is illustrated in Fig. 2. The graph representation of this configuration is shown in Fig. 3; it falls within the dependent group case and corresponds to values of $\alpha < .5$, namely, $\alpha = 1/N$.

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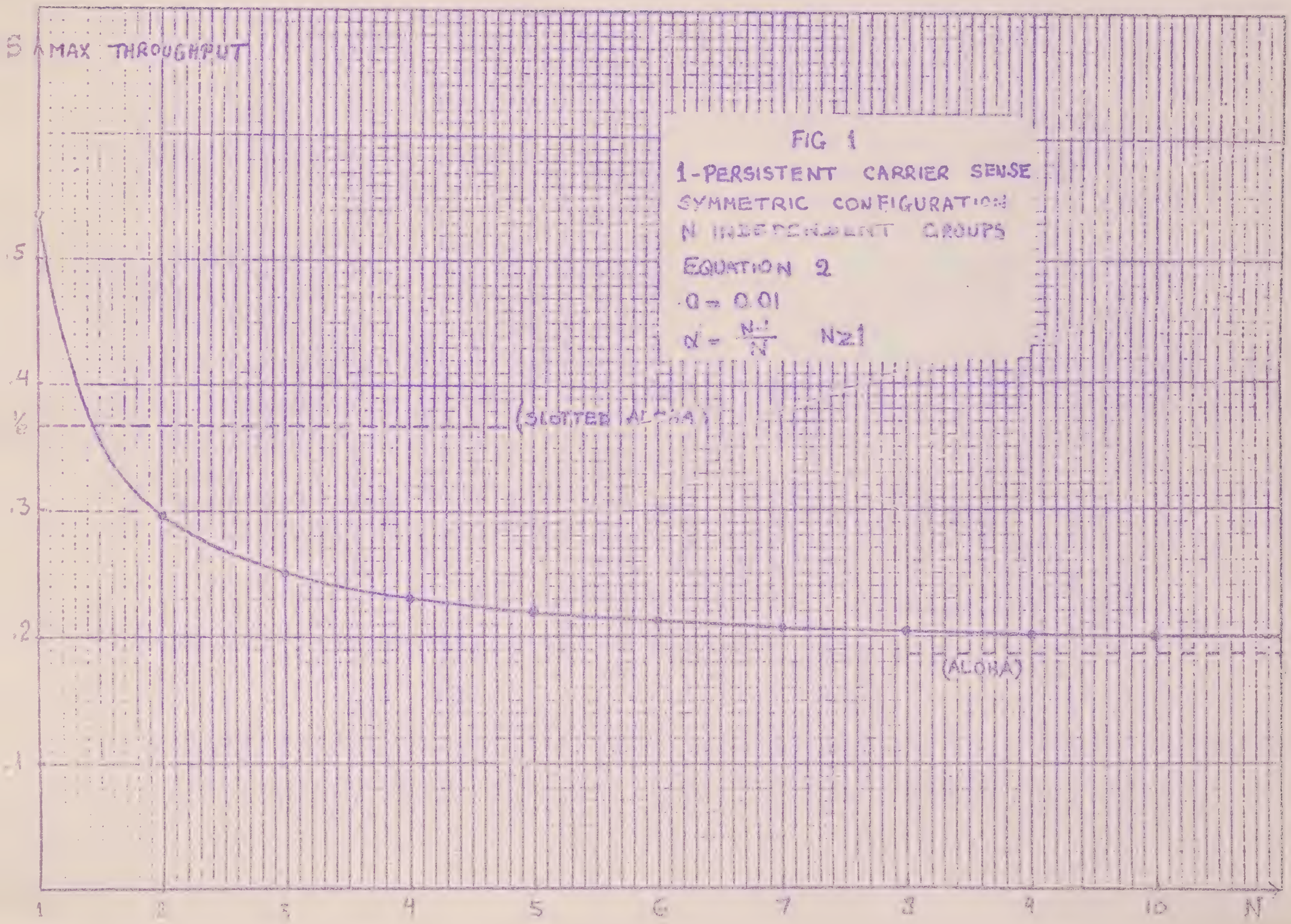

FIG 1

1-PERSISTENT CARRIER SENSE

SYMMETRIC CONFIGURATION

N INDEPENDENT GROUPS

EQUATION 2

$a = 0.01$

$\alpha = \frac{N-1}{N}$ $N \geq 1$

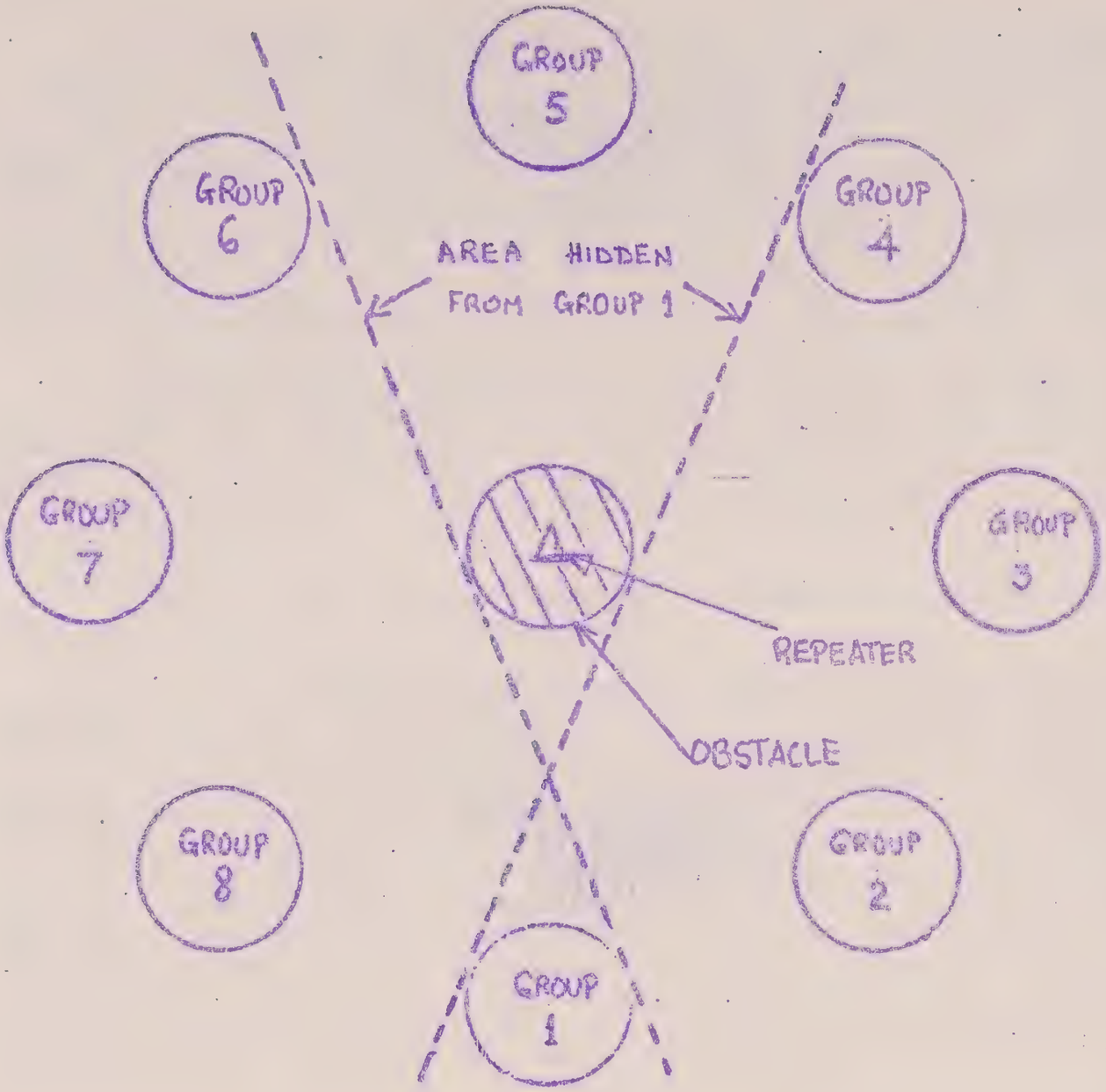
GROUP 5
GROUP 6
GROUP 4
AREA HIDDEN FROM GROUP 1
GROUP 7
GROUP 3
REPEATER
OBSTACLE
GROUP 8
GROUP 2
GROUP 1

FIG 2

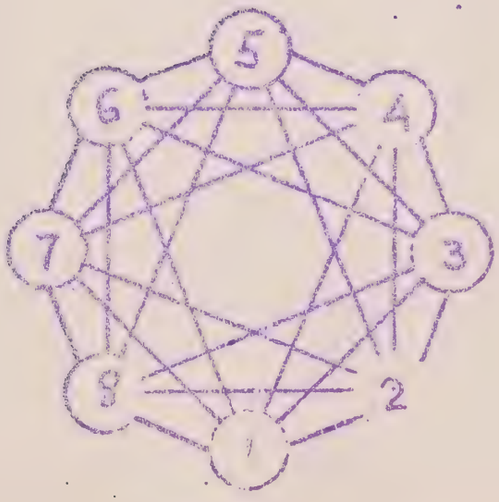
5
6
4
7
3
8
2
1

FIG 3

Simulation techniques have been used and the results are displayed in Figs. 4, 5, and 6. In Fig. 4 we show the relationship between the total throughput S and the total traffic G for various values of α (0.01, 0.05, 0.25, and 0.5). The higher the value of α is, the smaller is the maximum throughput achievable. This maximum is plotted against α in Fig. 5. Fig. 5 shows a sharp decrease in maximum throughput as α increases from 0, a rather unfortunate behavior. This shows the importance of considering the hidden terminal problem. From Fig. 4, we further note that for a given achievable throughput S, the larger α is, the larger the traffic rate G is and hence the smaller the probability of success of a packet is.

In Fig. 6, we plot minimum delay versus throughput for various values of α. The delay is minimized in the sense that simulation experiments were conducted in order to find the optimum retransmission delay X. Delays are heavily affected by the existence of hidden terminals. For example, for $S = .3$, the delay when $\alpha = .25$ is more than 5 times the delay when there are no hidden terminals.

(iii) A third instance which falls within the symmetric configuration category is described in the following.

The population of terminals is uniformly distributed only over an annular ring, the interior of the internal circle being empty. The repeater is at the center of the circle (Fig. 7). All terminals have the same range, and the combined power of simultaneously transmitted signals is assumed to have no effect on the range of carrier detection. Thus, if the range of the terminals is smaller than $2R$, then an arbitrary terminal T can only hear terminals whose range includes T and therefore the hidden terminal problem exists. To study a continuous problem such as this, we examine the discrete problem and break up the uniform distribution of terminals into small groups. We defer discussion of this technique to Section 4 in which we consider the wall-configuration. This example belongs to the dependent group case and has to be studied by simulation; however, it is just given as an example of the a-symmetric configuration, and there are no numerical results available.

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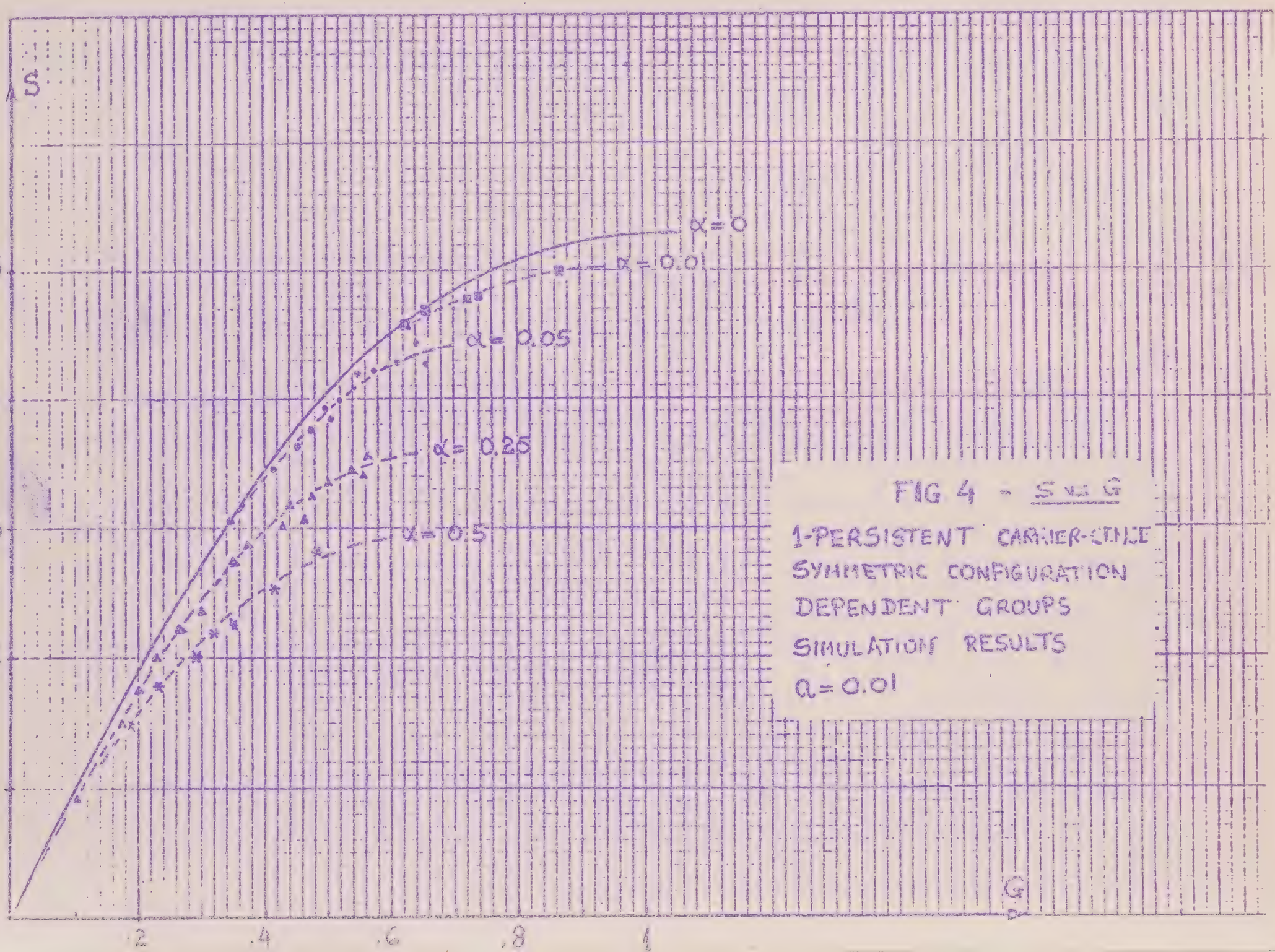

FIG 4 - S vs G

1-PERSISTENT CARRIER-SENSE
SYMMETRIC CONFIGURATION
DEPENDENT GROUPS
SIMULATION RESULTS
a = 0.01

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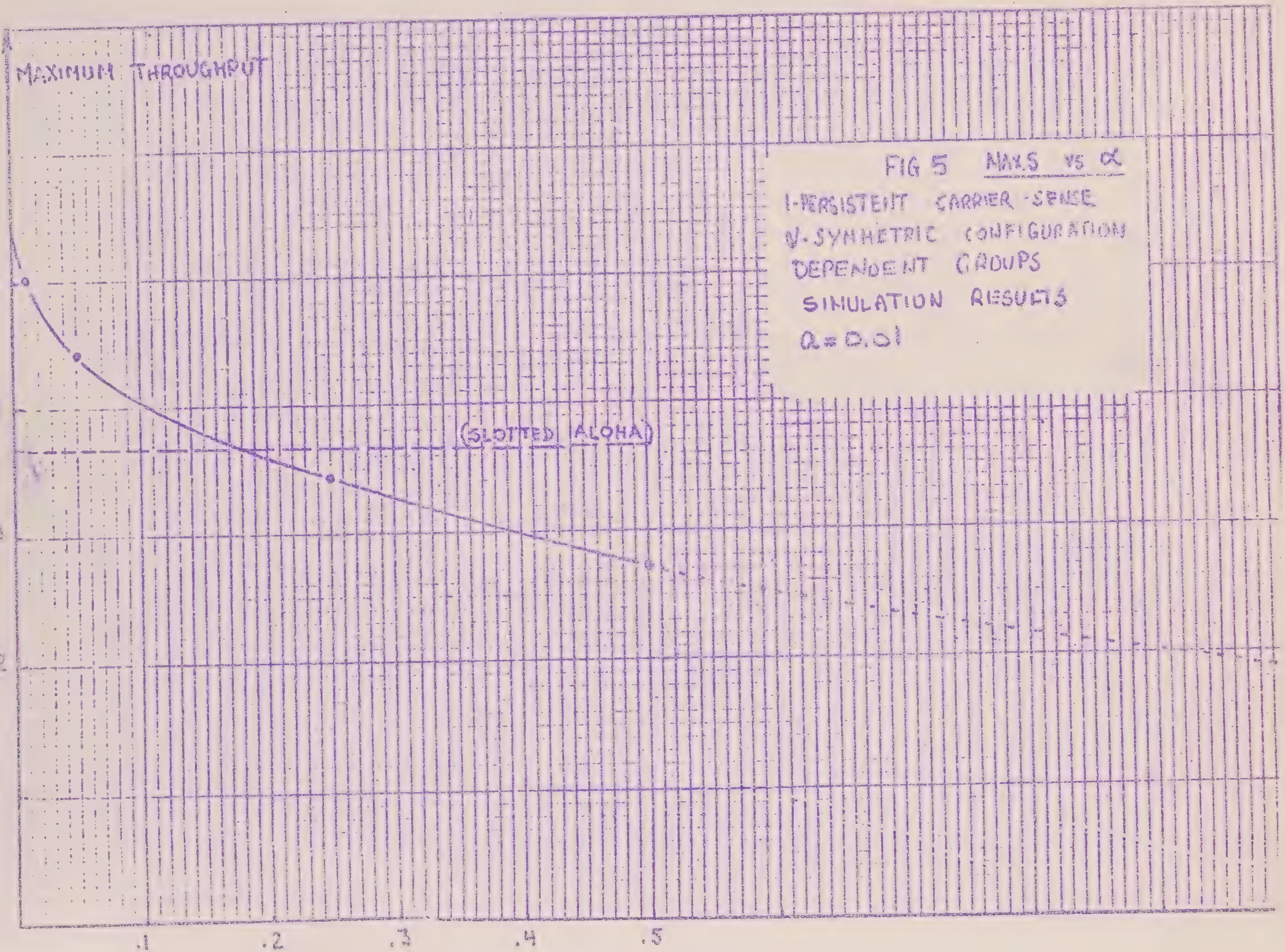

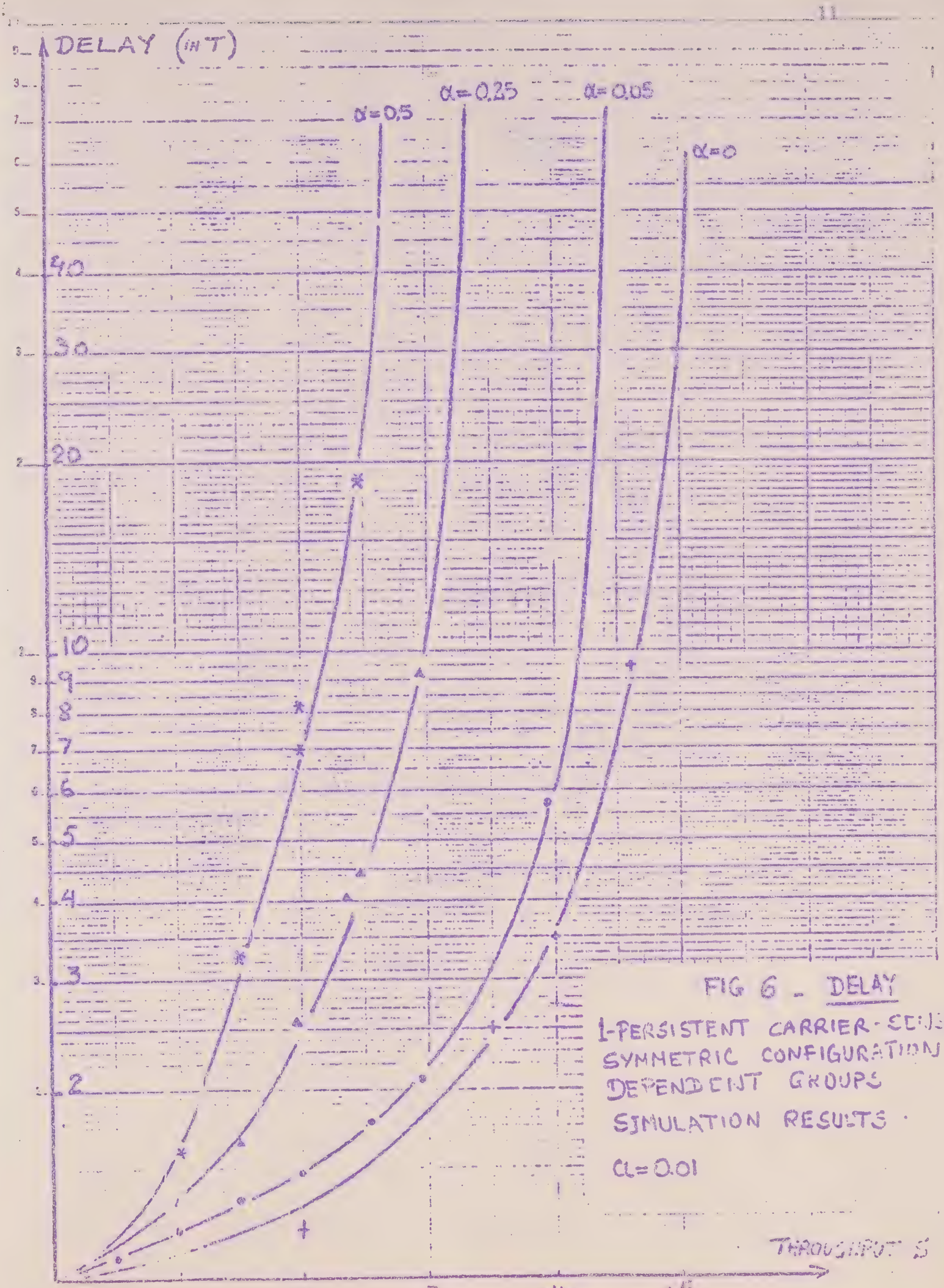

FIG 6 - DELAY

1-PERSISTENT CARRIER-SENSE
SYMMETRIC CONFIGURATION
DEPENDENT GROUPS
SIMULATION RESULTS.

a = 0.01

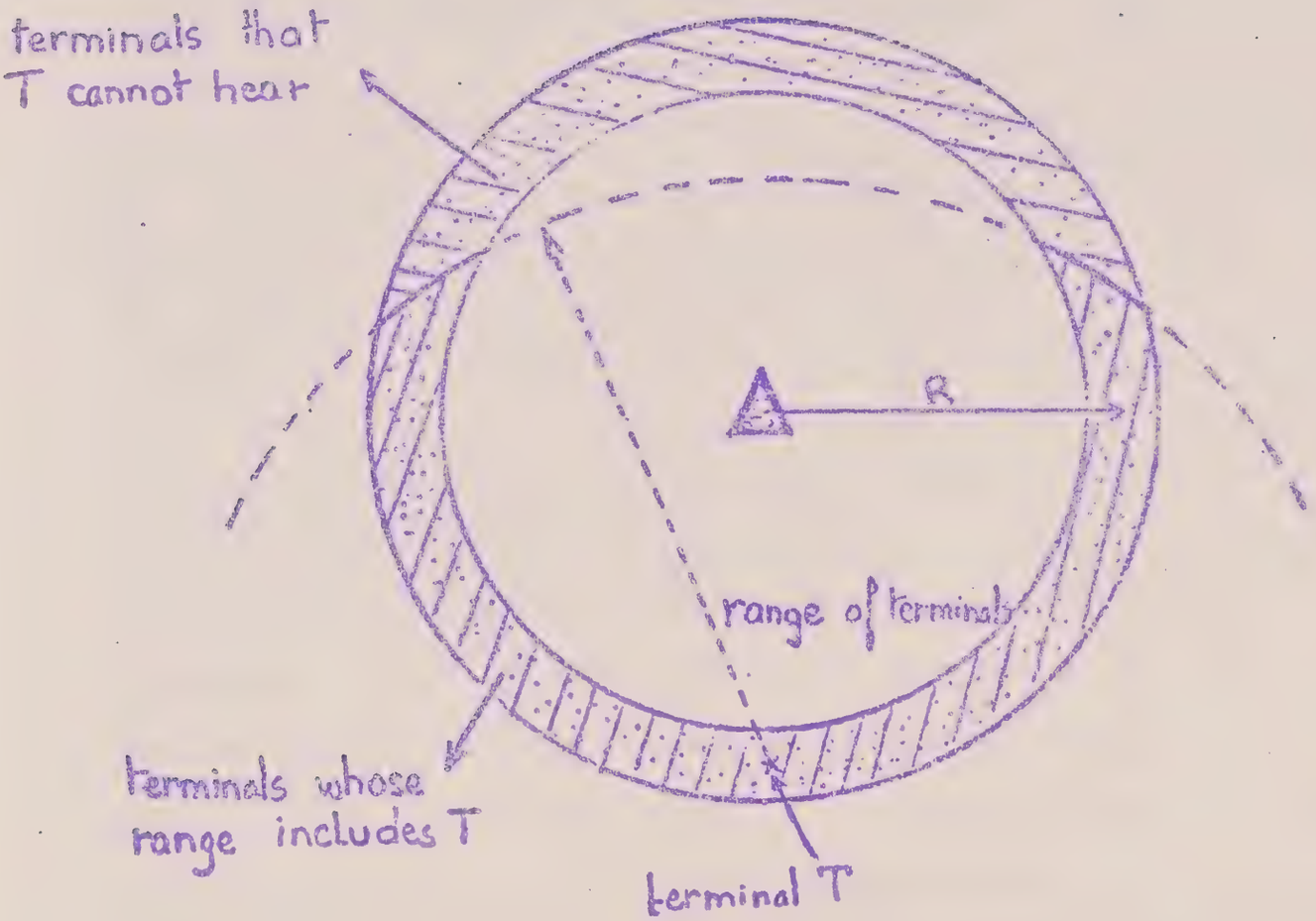
terminals that
T cannot hear
R
range of terminal
terminals whose
range includes T
terminal T

FIG. 7

b. Complementary Couple Configuration

Here, the population consists of 2 independent groups of unequal sizes. Let S and G denote the aggregate input rate and aggregate traffic rate respectively. S_1, G_1 and S_2, G_2 correspond to group 1 and 2 respectively. Let*

$$S_1 = \alpha S$$

$$S_2 = (1 - \alpha)S$$

Theorem 1 is readily applicable. All results are verified, with satisfaction, by simulation. We plot in Fig. 8 the throughputs S_1 and S_2 versus the traffic rates G_1 and G_2 for various values of α (0.001, 0.01, 0.1, 0.3, 0.5). These curves, obtained from Eq. 1, are compared to simulation results in Fig. 9. For each value of α, there exists a maximum achievable throughput S which we plot in Fig. 10. Here again we note that the maximum throughput decreases rapidly as α increases from 0. (This behavior is to be compared to the result obtained in example a(ii) above.) As soon as α is 10%, the 1-persistent carrier sense performs (throughput-wise) only as good as slotted Aloha. The feasible region in the S_1, S_2 plane is readily obtained from Fig. 10, and is shown in Fig. 11 along with the constant throughput contours ($S = S_1 + S_2 = \text{constant}$) and the constant α contours ($S_2 = \frac{1 - \alpha}{\alpha} S_1$). A good measure of delay can be obtained by considering the average number of transmissions per packet. We plot in Fig. 12 G_1/S_1 and G_2/S_2 against the total throughput S. We note that the larger group (the group with larger collective input rate) performs much better than the smaller one. Their relative performance is shown in Fig. 13 by plotting the ratio $\frac{G_1}{S_1} / \frac{G_2}{S_2}$ versus S for various values of α. The overall average number of transmissions $(\alpha \frac{G_1}{S_1} + (1 - \alpha) \frac{G_2}{S_2})$ is shown in Fig. 14. Although we noted that for $\alpha = 10\%$, 1-persistent carrier sense performs (thoughput-wise) only as good as slotted Aloha, we see here that, for the same value of α, the average number of transmission is much lower for the 1-persistent carrier sense. Finally, we note that, as far as both throughput and number of transmissions are concerned, the more balanced the configuration is (i.e., the closer α is to .5), the worse the performance of the 1-persistent access mode is.

4. Non-Persistent Carrier Sense

Similar to the previous section, we define here for each group a time axis on which we observe an alternate sequence of busy and idle periods. The simplicity in studying the non-persistent carrier sense is mainly due to the fact that any busy period consists of a single transmission period. We first consider the independent group case and then proceed to the general case. The treatment and discussion of some examples is deferred to Section 4.3.

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*α here is the fraction of the total population contained in group 1.

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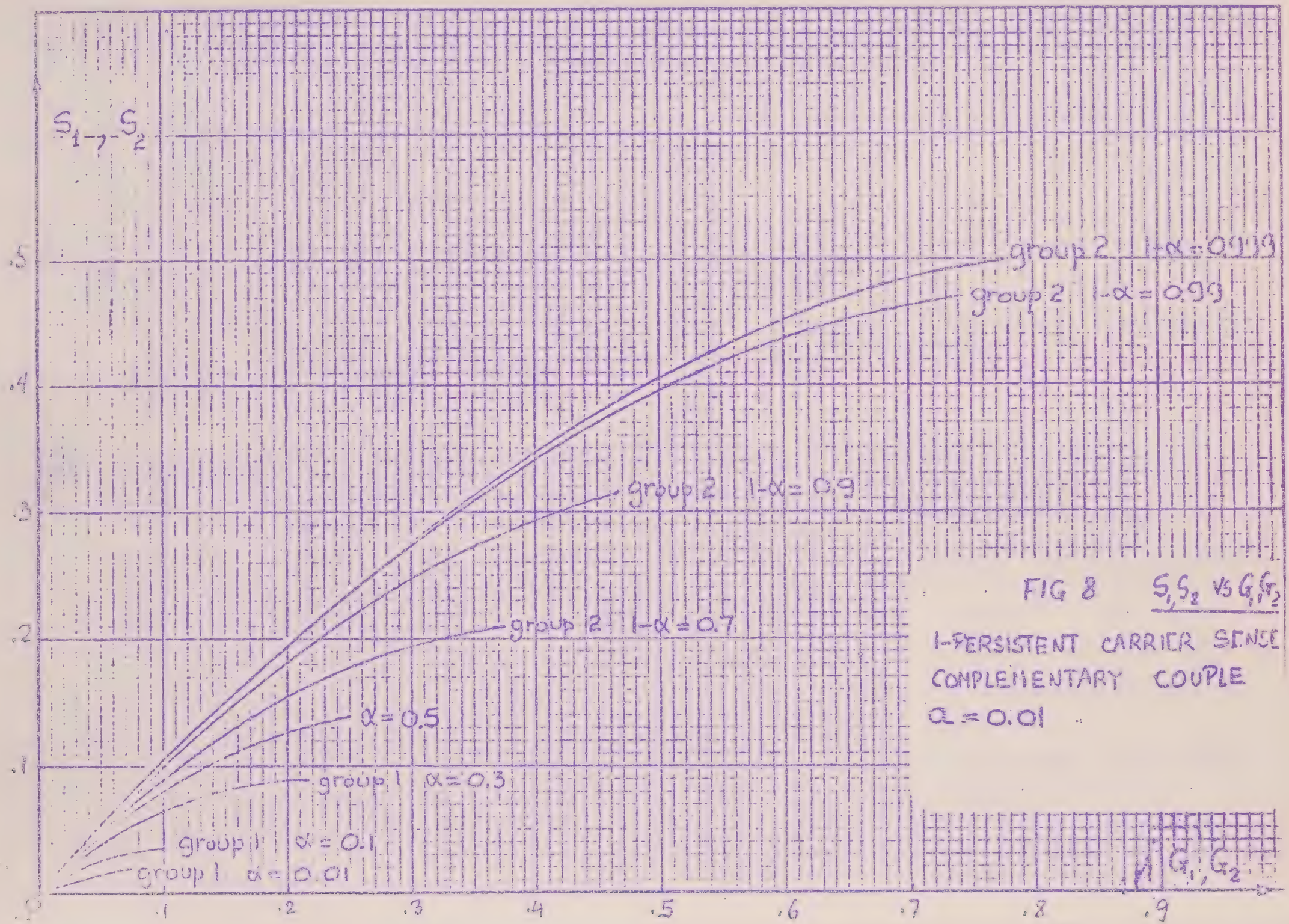

FIG 8 S_1, S_2 VS G_1, G_2

1-PERSISTENT CARRIER SENSE

COMPLEMENTARY COUPLE

$a = 0.01$

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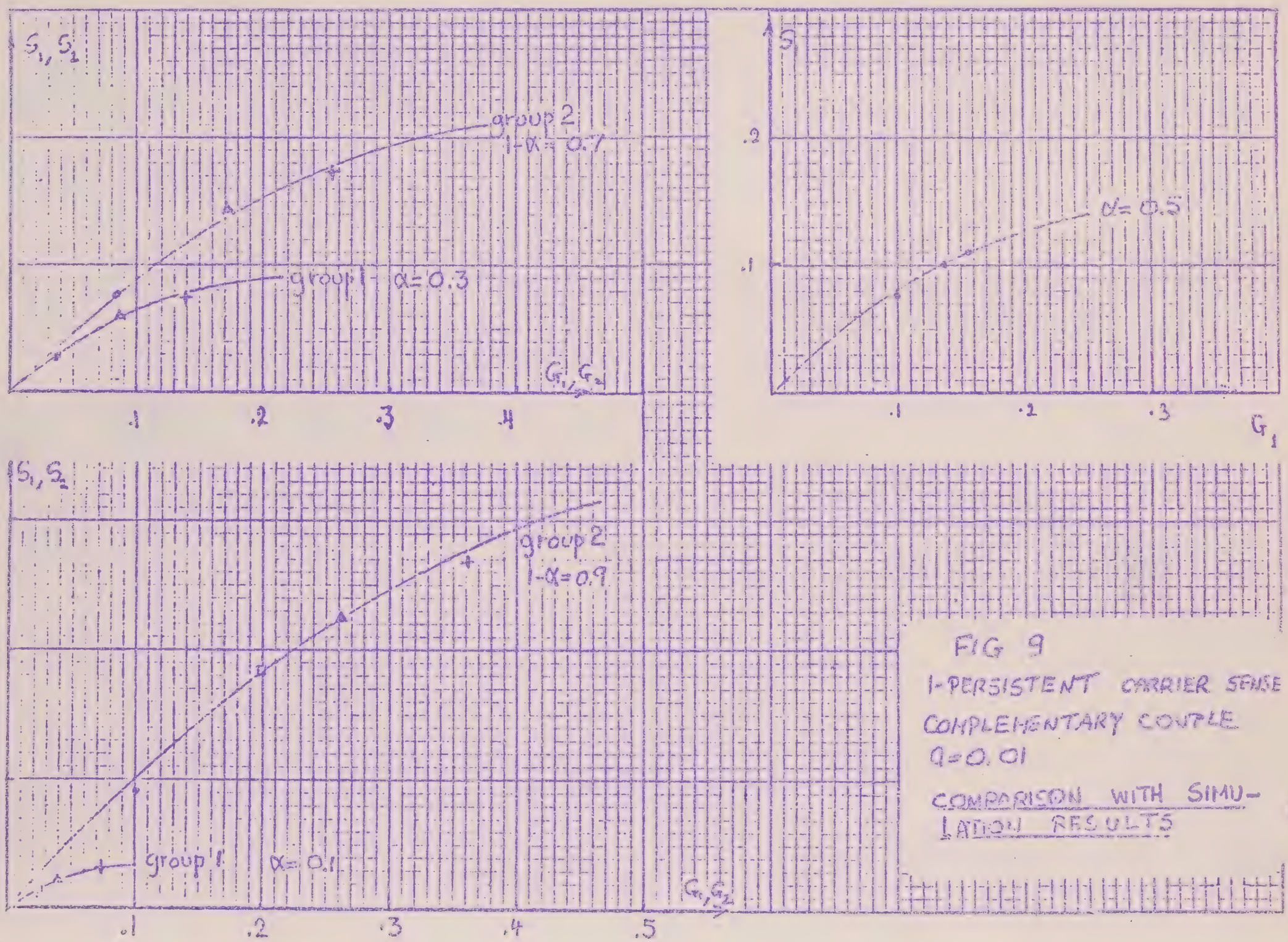

FIG 9

1-PERSISTENT CARRIER SENSE

COMPLEMENTARY COUPLE

$a = 0.01$

COMPARISON WITH SIMULATION RESULTS

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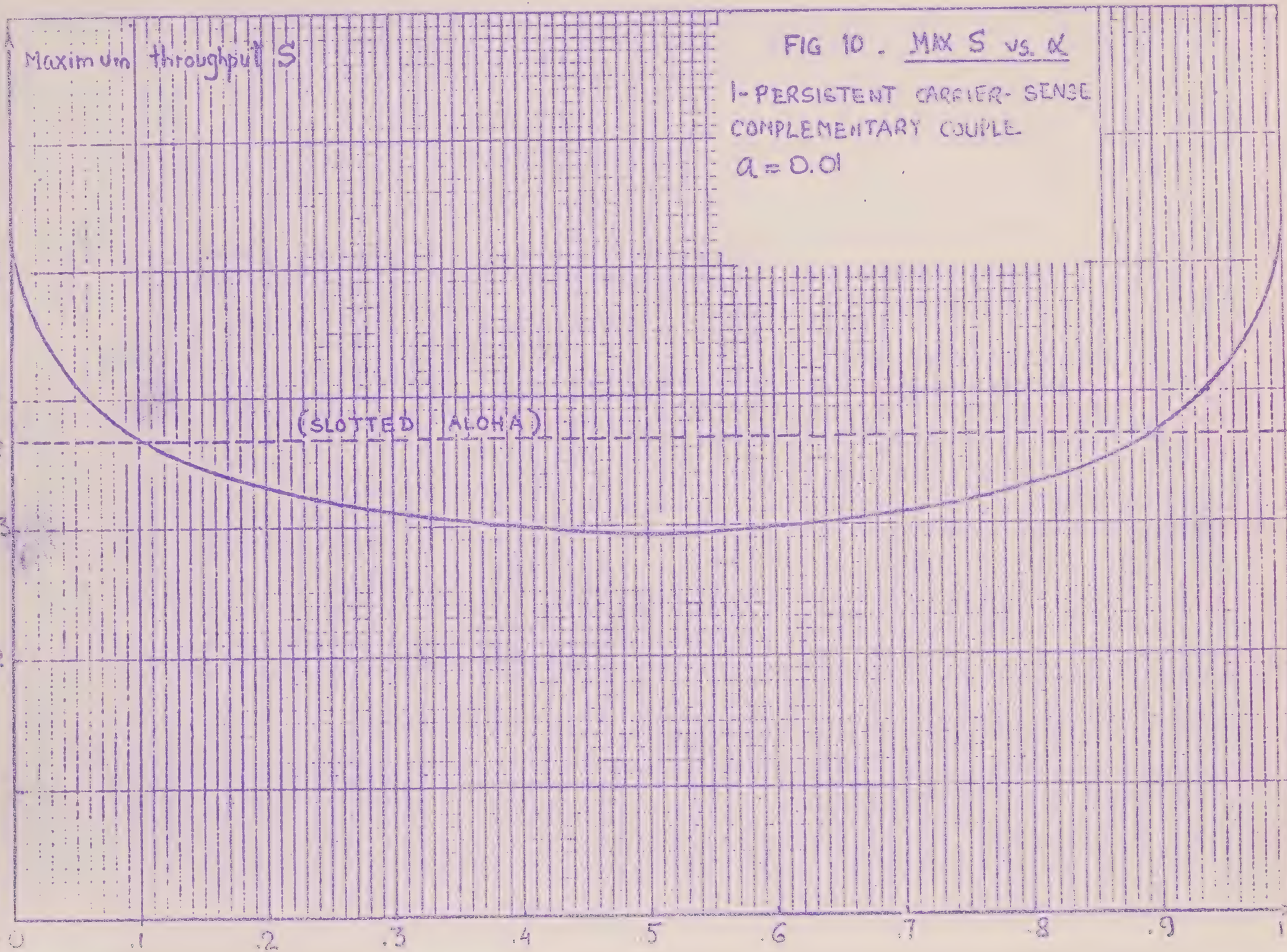

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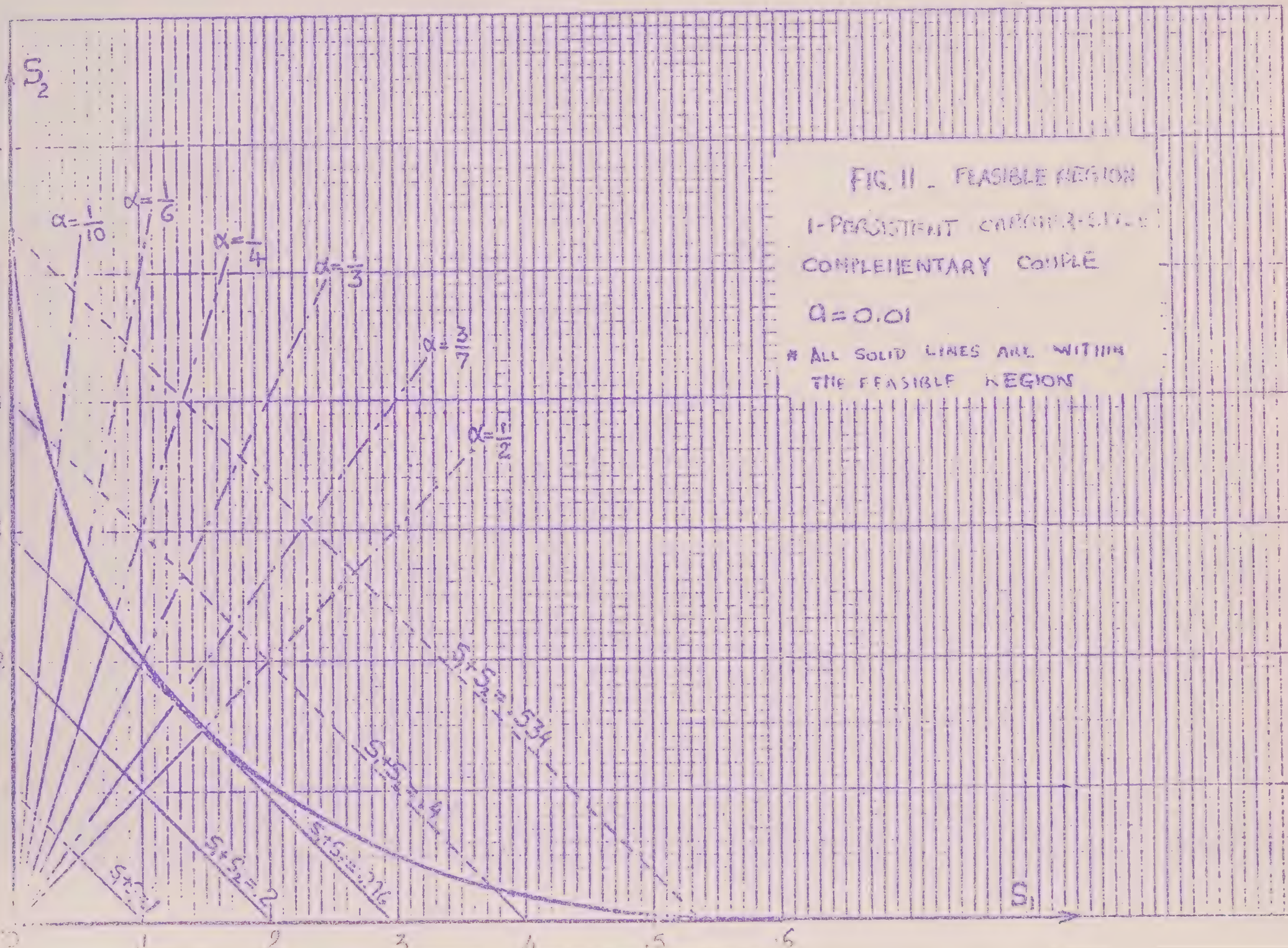

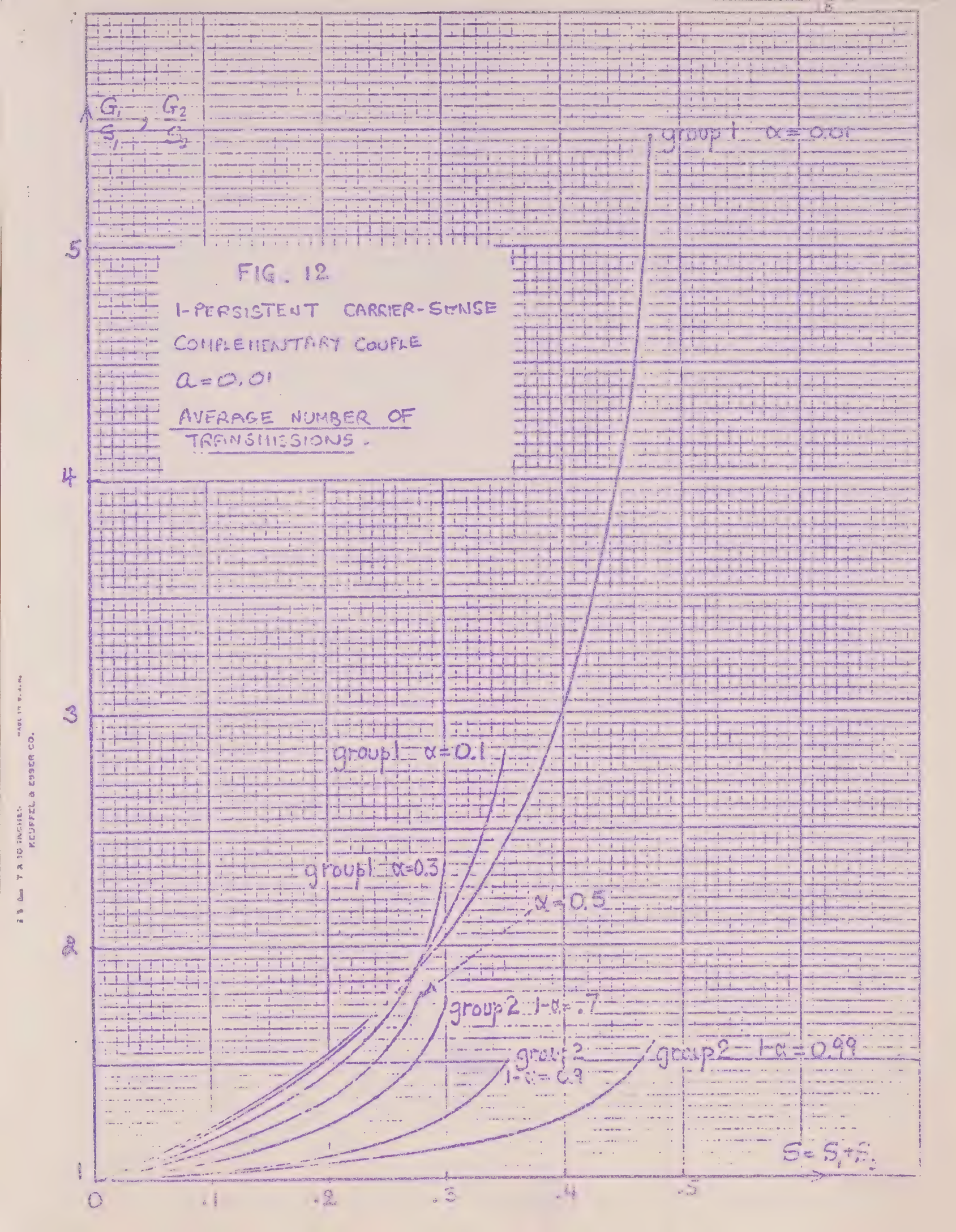

FIG. 12

1-PERSISTENT CARRIER-SENSE COMPLEMENTARY COUPLE

$a = 0.01$

AVERAGE NUMBER OF TRANSMISSIONS.

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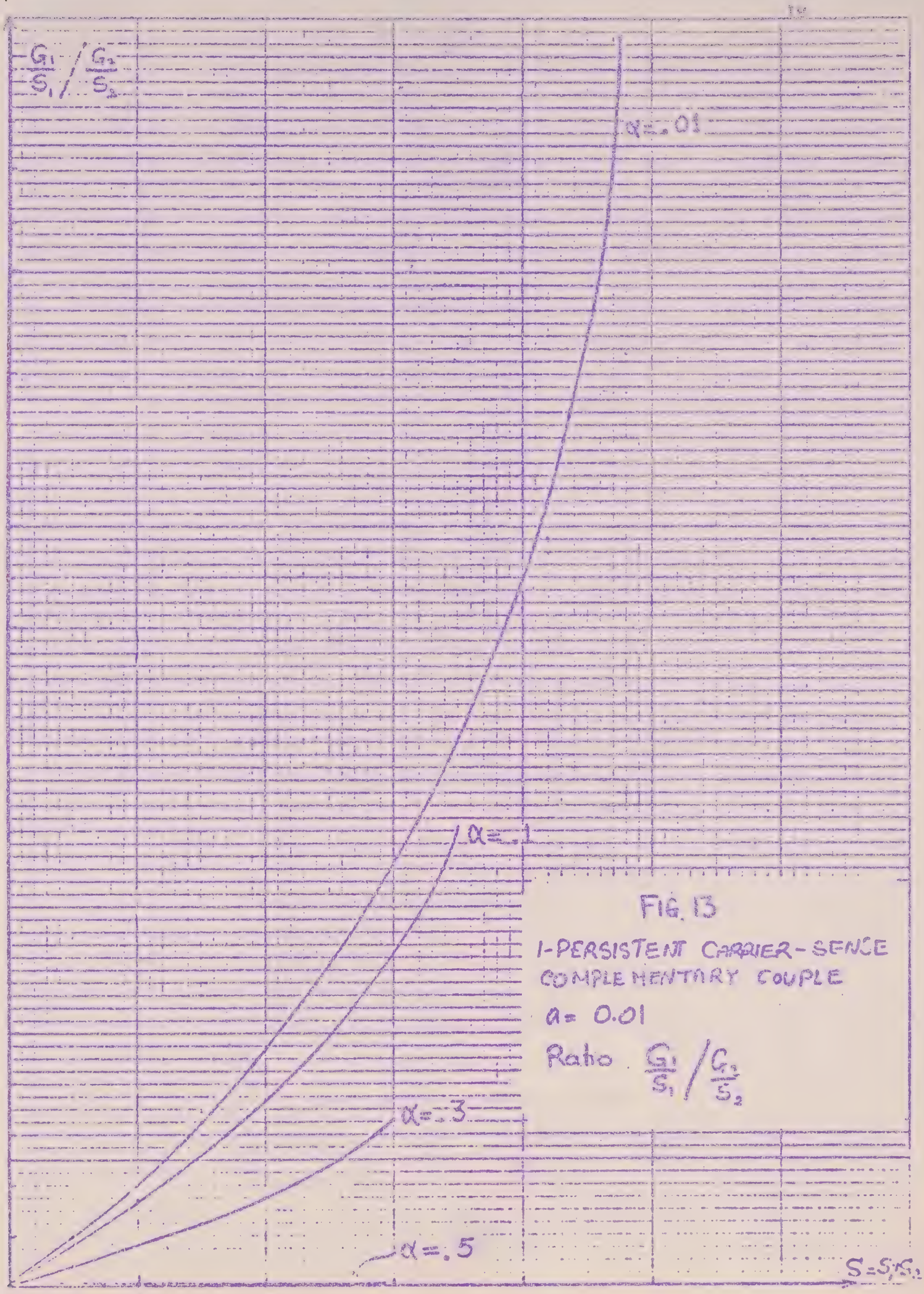

FIG. 13

1-PERSISTENT CARRIER-SENCE COMPLEMENTARY COUPLE

a= 0.01

Ratio $\frac{G_1}{S_1} / \frac{G_2}{S_2}$

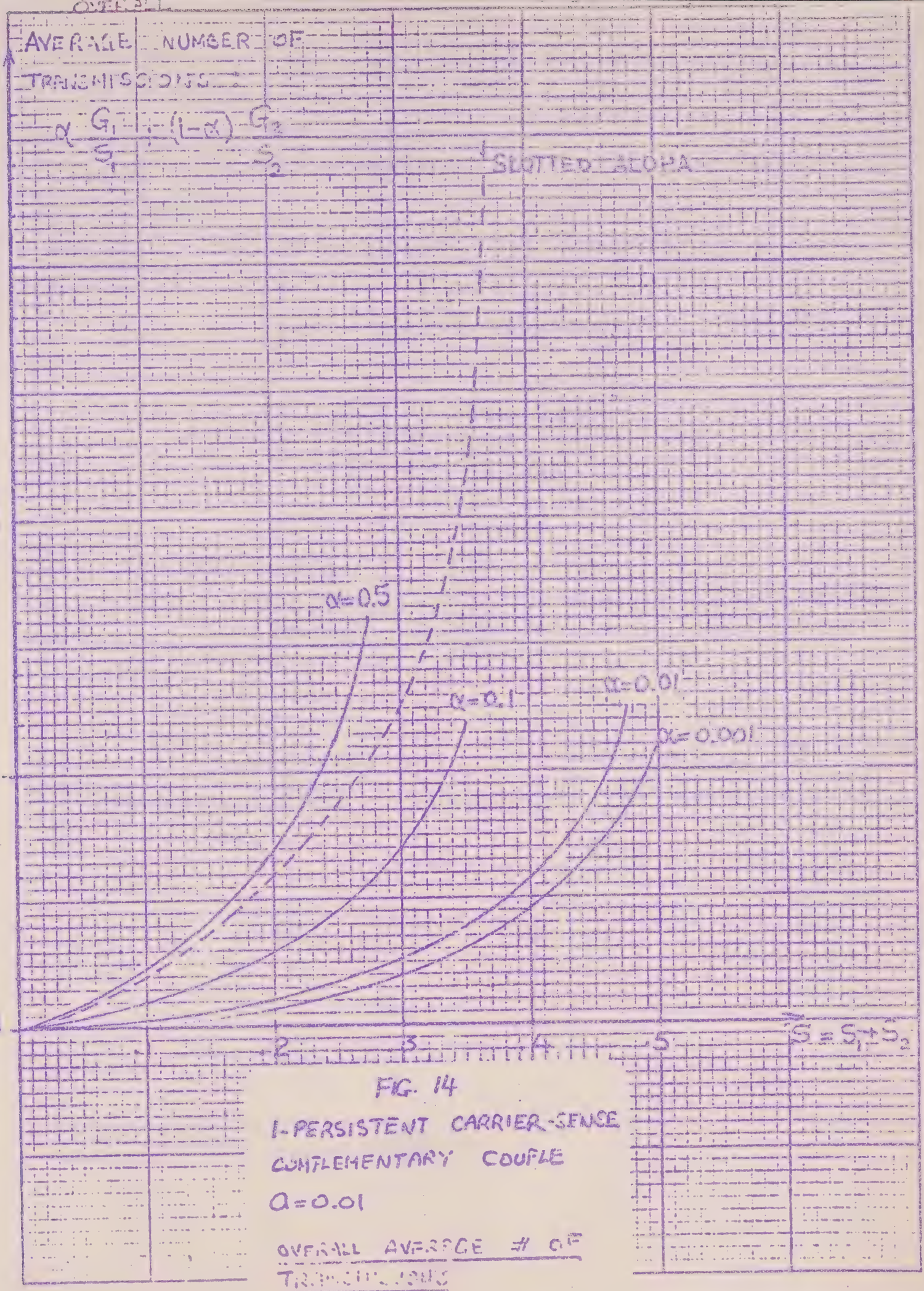

FIG. 14

1-PERSISTENT CARRIER-SENSE COMPLEMENTARY COUPLE

$a = 0.01$

OVERALL AVERAGE # OF TRANSMISSIONS

4.1 Independent Groups

Let S_i denote the input rate for group i, G_i the rate of its look point process.

Assumption A'1: The input process and the look point process are both independent Poisson processes. The propagation delay τ is identical for all terminals.

Theorem 2.* Under assumption A'1, the probability of success of a packet from group i is given by:

$$P_{S_i} = \frac{S_i}{G_i} = \frac{e^{-aG_i}}{(1 + aG_i)e^{-G_i}} \prod_{j=1}^{N} \frac{(1 + aG_j)e^{-G_j}}{G_j(1 + 2a) + e^{-aG_j}} \qquad (2a)$$

4.2 Dependent Group Case

Focusing on line i, denote by B_i the renewal point process defined by the starting point of busy periods (Fig. 15) and let H_i denote the rate of process B_i.

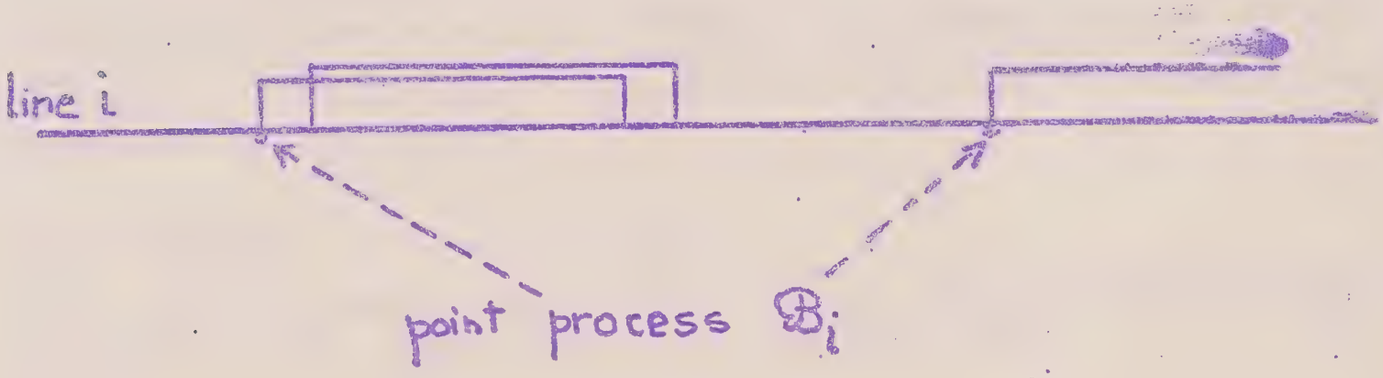

FIG. 15.

*See Appendix B for proof of this theorem.

Assumption A'2: The process S_i (rate H_i) $i = 1, \ldots, N$, are mutually independent processes. (We recognize that this statement is not true. However, it is used in the derivation in order to find an approximate relationship between the rates of various processes.)

Theorem 3.* Under the additional assumption A'2, the relationship between the set $\{H_i\}$ and the set $\{G_i\}$ is given by the following system of equations:

$$H_i = G_i[1 - H_i(1 + \bar{y}_i + a)] \prod_{\substack{j \in h(i) \\ j \neq i}} \left\{[1 - H_j(1 + \bar{y}_j)]\right\} \quad i = 1, 2, \ldots, N \qquad (3)$$

Theorem 4.* Under the above assumptions the throughput of group i is given by

$$S_i = G_i \prod_{j \in h(i)} \left\{[1 - H_j(1 + \bar{y}_j + a)]e^{-aG_j}\right\} \prod_{k \notin h(i)} \left\{[1 - H_k(1 + \bar{y}_k)]e^{-G_k}\right\} \qquad (4)$$

Theorems 3 and 4 provide us with a system of 2N non-linear equations relating the sets of variables $\{S_i\}$, $\{G_i\}$ and $\{H_i\}$. Given an arbitrary configuration and a set of input rates $\{S_i\}$, the system is solved iteratively, giving answers to questions Q1 and Q2.

4.3 Examples

a. Here again we consider the case of n independent groups of equal size. Let $H_i = h$, $G_i = g$, and $S_i = s$. Theorem 2 gives:

$$P_S = \frac{S}{G} = \frac{e^{-ag}}{(1 + ag)e^{-g}} \left(\frac{(1 + ag)e^{-g}}{g(1 + 2a) + e^{-ag}}\right)^N \qquad (5)$$

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*See Appendix B for proof.

Letting $N \to \infty$, s, $g \to 0$ such that $N_s = S$ and $N = G$, we can easily see that

$$P_S = \frac{S}{G} = e^{-2S}$$

Applying theorems 3 and 4 we have

$$h = g(1 - h(1 + \bar{y} + a))$$

$$s = g(1 - h(1 + \bar{y} + a))\ (1 - h(1 + \bar{y}))^{N-1} e^{-(N-1)g - ag}$$

After eliminating h from the two above equations, we get

$$\frac{s}{g} = \frac{S}{G} = \frac{e^{-ag}}{(1 + ag)e^{-g}} \left[\frac{(1 + ag)e^{-g}}{g(1 + 2a + e^{-ag}} \right]^N$$

which is identical to Eq. 5.

b. The Wall-Configuration

Consider a uniform distribution of terminals over a circular area, the repeater being located at the center. All terminals are within range, but the presence of a wall (hill) as displayed in Fig. 17 causes the existence of the hidden terminal problem: A terminal T at an angle α_0 ($\alpha_0 < 180°$) from the wall can only hear terminals in the region defined by $0 < \alpha < \alpha_0 + 180°$. To study a continuous problem such as this, we examine the discrete problem and break up the uniform population into small sectors; each sector is then considered as a group. The smaller the sectors are, the more precise the approximation becomes.

The problem is solved numerically for $N = 5$ and $N = 10$ below.

5 Groups discrete approximation:

The graph representation considered for this case is shown in Fig. 18. Theorems 3 and 4 are applied. Iteration results are compared to simulation results in Table 1. The agreement is fairly satisfactory; as already observed (see for example Fig. A-2), the simulation reaches saturation*

*From a simulation point of view, the channel is said to be saturated when the backlog of packets exceeds a given threshold, the latter being set by the limitations of the program.

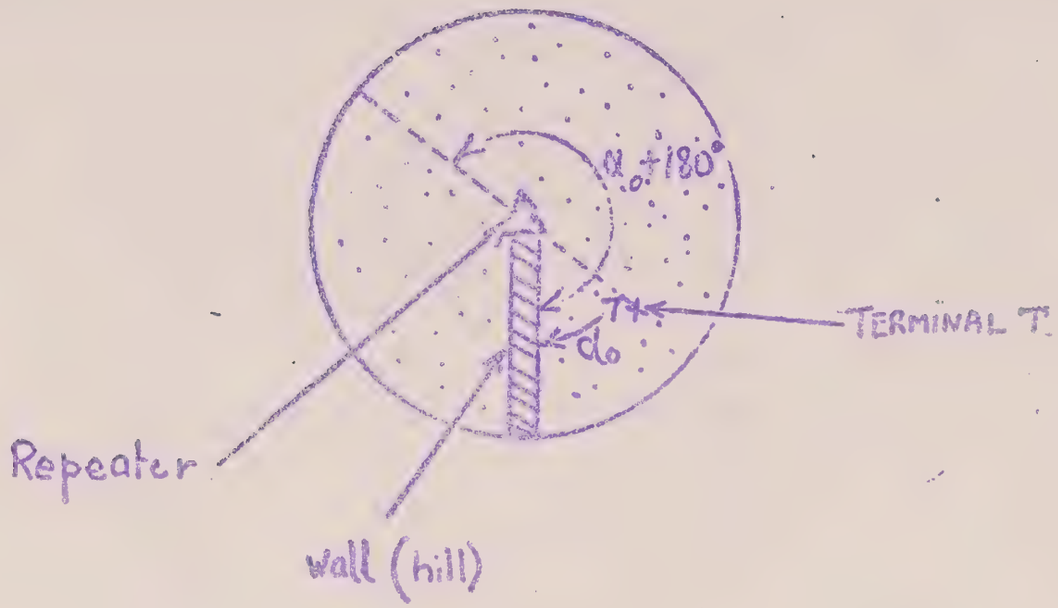
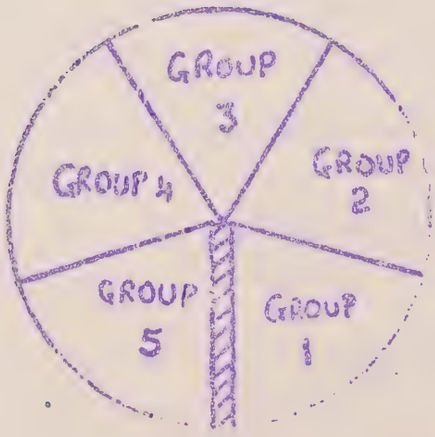


FIG. 17 WALL-CONFIGURATION



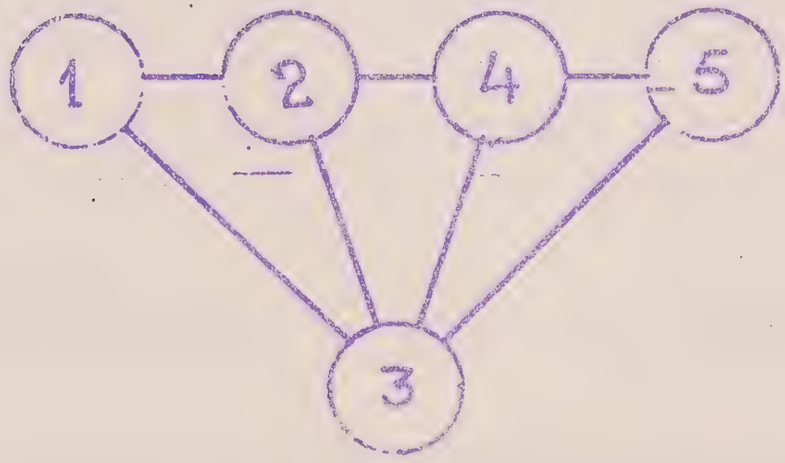


FIG. 18 5 SECTORS

for an input rate lower than the maximum predicted by the analytic model. Also, when the input rate is close to saturation, the look point rate G_i' provided by simulation is larger than that provided by the model. This is due to the instability of such random access modes.

TABLE 1

S	G1	G'1	G2	G'2	G3	G'3
0.1	0.023		0.023		0.022	
* 0.2	0.057	0.057	0.054	0.054	0.051	0.0519
* 0.25	0.080	0.084	0.074	0.076	0.065	0.069
* 0.28	0.097	0.105	0.089	0.094	0.081	0.0836
* 0.3	0.111	0.128	0.101	0.112	0.090	0.097
0.31	0.119		0.107		0.095	
* 0.315		0.152		0.129		0.110
0.32	0.128		0.114		0.101	
0.35	0.161		0.140		0.120	
0.39	0.273		0.220		0.168	

S groups discrete approximation

S total input rate

G_i look point rate for group i (iteration results)

G_i' look point rate for group i (simulation results)

a = 0.01

* input rate for which simulation results are available

---

10 groups discrete approximation

To get a better approximation we increase the number of sectors to 10.

Upper and lower bounds

For each group i, there exists a group $\bar{I}$ diametrically opposite. A terminal in group i can only hear a fraction of group $\bar{I}$. The solution to the continuous problem is therefore bounded by the two cases :

Case 1:

$\bar{I}_{ch}(i)$ which corresponds to a lower bound on maximum throughput or upper bound on G and on the average number of transmissions. The matrix representation for this case is

	1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	0	0	0	0	0
2	1	1	1	1	1	1	0	0	0	0
3	1	1	1	1	1	1	1	0	0	0
4	1	1	1	1	1	1	1	1	0	0
5	1	1	1	1	1	1	1	1	1	0
6	0	1	1	1	1	1	1	1	1	1
7	0	0	1	1	1	1	1	1	1	1
8	0	0	0	1	1	1	1	1	1	1
9	0	0	0	0	1	1	1	1	1	1
10	0	0	0	0	0	1	1	1	1	1

Case 2:

$\bar{I}_{ch}(i)$ which corresponds to an upper bound on maximum throughput or lower bound on G and on the average number of transmissions.

	1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	1	1	1	0	0	0
3	1	1	1	1	1	1	1	1	0	0
4	1	1	1	1	1	1	1	1	1	0
5	1	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1	1
7	0	1	1	1	1	1	1	1	1	1
8	0	0	1	1	1	1	1	1	1	1
9	0	0	0	1	1	1	1	1	1	1
10	0	0	0	0	1	1	1	1	1	1

For case 1, iteration results and simulation results are compared in Table 2 ($a = 0.01$, $a = 0.1$). This again validates the iterative model provided by theorems 3 and 4 (for larger values of a than 0.01).

TABLE 2

a	S	Gi	G'1	G2	G'2	G3	G'3	G4	G'4	G5	G'5
	0.1	0.012	0.0116	0.012	0.012	0.012	0.0113	0.011	0.0117	0.011	0.0113
	0.15	0.02	0.02	0.019	0.0195	0.019	0.019	0.019	0.0175	0.018	0.018
0.01	0.2	0.029	0.029	0.020	0.028	0.028	0.028	0.027	0.027	0.026	0.026
	0.23	0.036	0.039	0.035	0.037	0.034	0.036	0.033	0.035	0.032	0.033
	0.26	0.045	0.048	0.043	0.047	0.042	0.044	0.040	0.042	0.038	0.040
	0.29	0.056	0.063	0.054	0.060	0.051	0.056	0.049	0.052	0.049	0.050
	0.1	0.012	0.0115	0.012	0.012	0.012	0.0113	0.012	0.0117	0.012	0.0117
	0.15	0.02	0.02	0.02	0.0195	0.019	0.0193	0.019	0.018	0.019	0.0188
0.1	0.175	0.025	0.0245	0.024	0.0245	0.024	0.0235	0.023	0.0235	0.023	0.0232
	0.2	0.03	0.030	0.03	0.030	0.029	0.029	0.028	0.028	0.028	0.0275
	0.225	0.037	0.037	0.036	0.0365	0.035	0.036	0.034	0.036	0.033	0.0345
	0.25	0.045	0.046	0.044	0.0455	0.042	0.0433	0.041	0.042	0.039	0.0405
	0.275	0.056	0.072	0.053	0.065	0.051	0.065	0.049	0.06	0.047	0.057

10 groups discrete approximation

S total input rate

G_i look point rate for group i (iteration results)

G'_i look point rate for group i (simulation results)

Iteration results for both cases 1 and 2 are plotted in Figs. 20, 21, and 22 ($a = 0.01$). In Fig. 20 we plot the total throughput S against the aggregate rate G. The maximum throughput S is bounded by the two cases, i.e.,

$$.37 < S_{max} < .44 \qquad (a = 0.01)$$

FIG. 19 - 10 SECTORS

The existence of the wall decreased the maximum throughput by approximately a factor of 2. Similarly, in Fig. 21, we plot the upper and lower bound on overall average number of transmissions per packet.

In order to compare the relative performance of a terminal with respect to its position, we plot the upper and lower bounds on average number of transmissions as a function of the angle α:

- For low values of throughput, the difference in performance between the sectors is rather small, and the upper and lower bounds are fairly close. These differences get much larger as the input rate increases. For a more precise answer, the larger the input rate is, the larger the number of sectors should be.

- The more terminals a terminal can see, the better its performance is. This is similar to the behavior observed in the complementary couple example where the large group performs better than the smaller one.

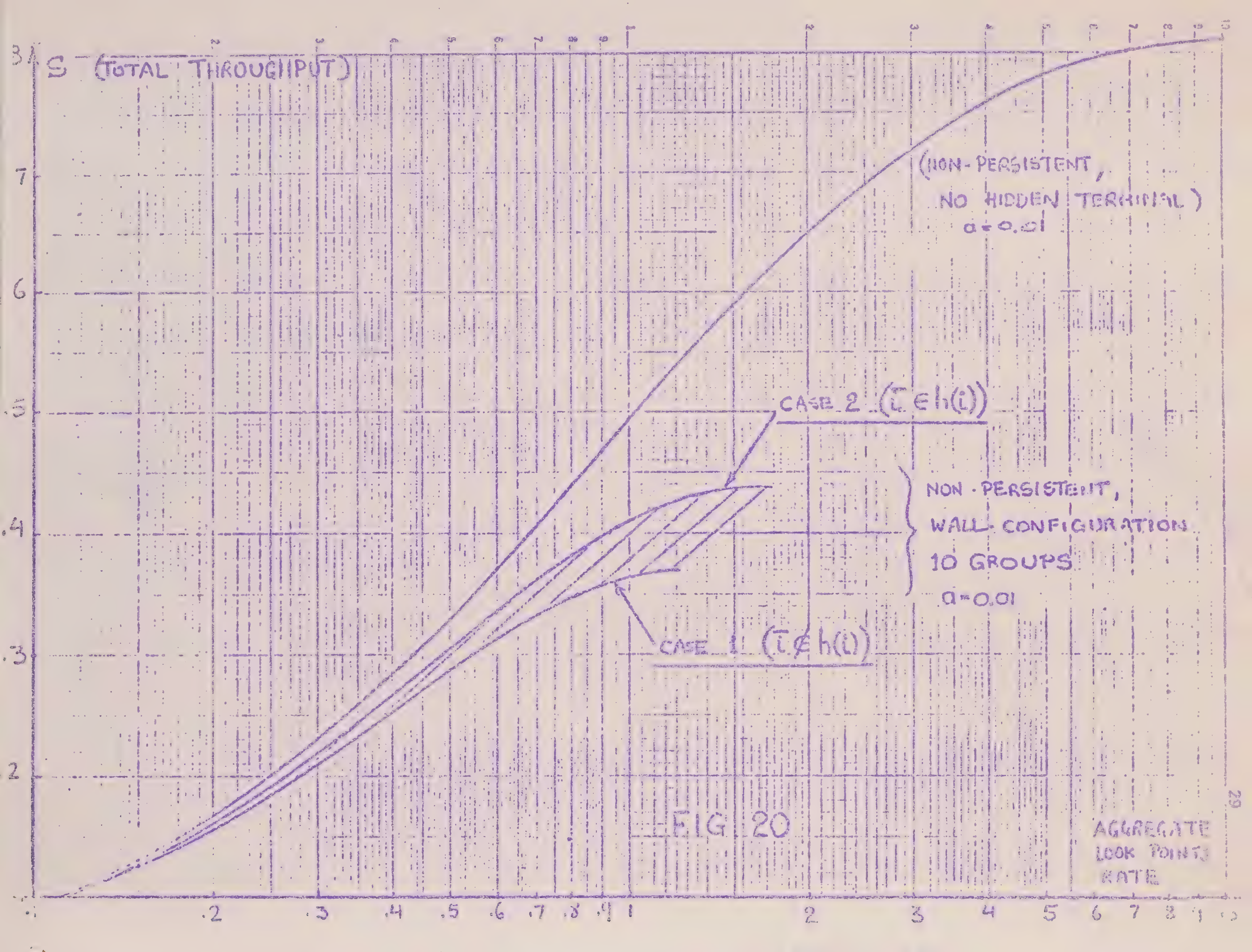
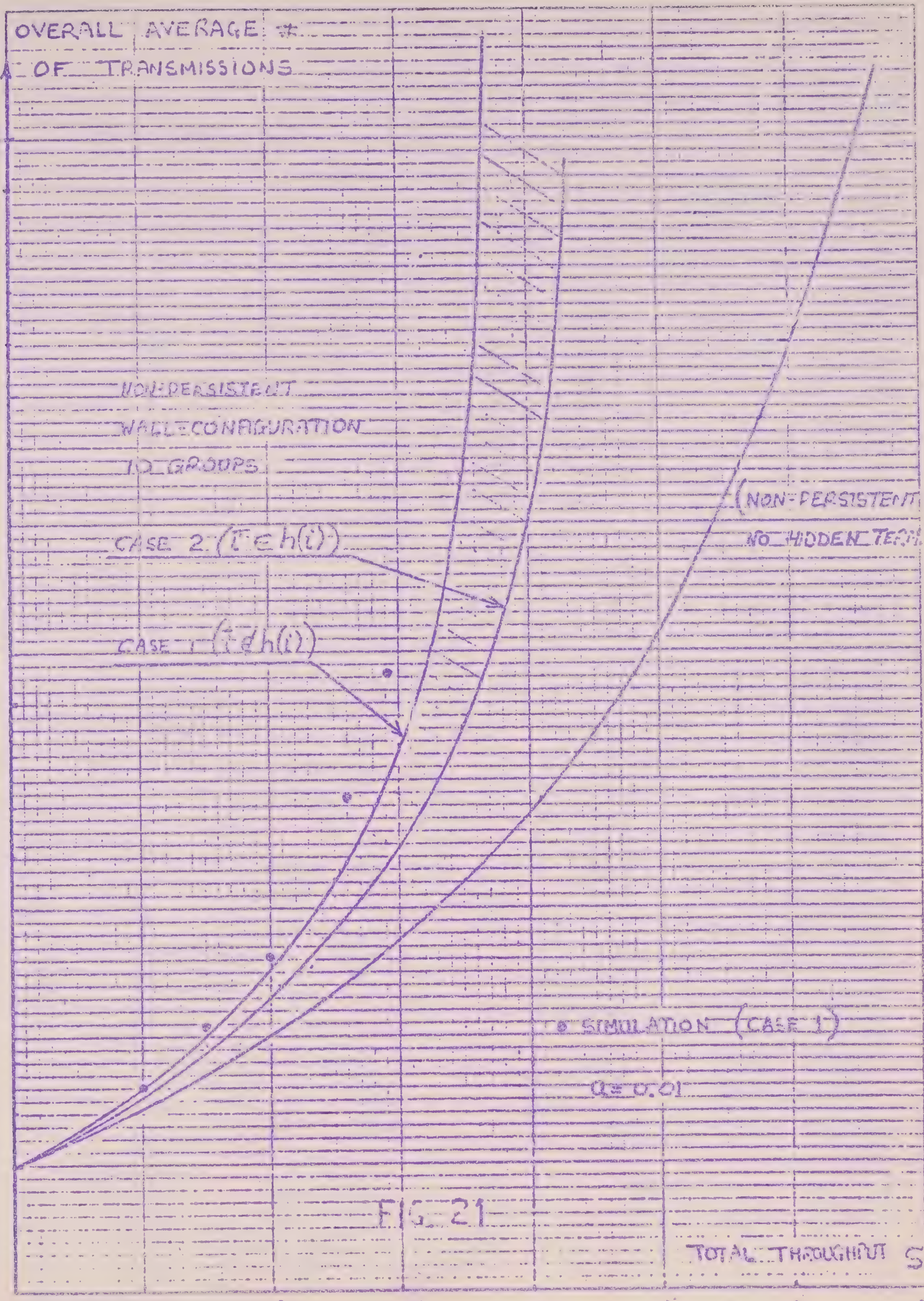

S (TOTAL THROUGHPUT)
(NON-PERSISTENT, NO HIDDEN TERMINAL)
a=0.01
CASE 2 (i ∈ h(i))
NON-PERSISTENT, WALL-CONFIGURATION 10 GROUPS
a=0.01
CASE 1 (i ∉ h(i))
AGGREGATE LOOK POINTS RATE
.2
.3
.4
.5
.6
.7
.8
.9
1
2
3
4
5
6
7
8
9
10

FIG 20


OVERALL AVERAGE #
OF TRANSMISSIONS
3
2
1
NON-PERSISTENT
WALL CONFIGURATION
10 GROUPS
CASE 2 (t ∈ h(i))
CASE 1 (t ∉ h(i))
(NON-PERSISTENT
NO HIDDEN TERM)
SIMULATION (CASE 1)
Q = 0.01
FIG. 21
TOTAL THROUGHPUT S
.1
.2
.3
.4
.5
.6
.7

AVERAGE # OF TRANSMISSIONS
0
5
10
15
36°
72°
108°
144°
180°
216°
252°
288°
324°
360°
CASE 1
CASE 2
CASE 3
CASE 4
S=1
S=2
S=3
S=4
S=36

5. Concluding Remarks

A general representation of terminal configurations with hidden elements has been introduced; a distinction between dependent and independent groups has been made. Mathematical models are available for the independent group case in 1-persistent carrier sense and for the general case in non-persistent carrier sense. These models can be used to determine the performance of the access mode. All results were verified by simulation. Whenever analytic models are unavailable, simulation techniques were used. For the various particular configurations that were considered it is to be noted that the performance of carrier sense is badly degraded by the existence of hidden elements. In terms of average number of retransmissions, the more terminals a given terminal can hear, the better its performance is.

APPENDIX A

NON-PERSISTENT CARRIER SENSE

The non-persistent carrier sense access mode, also called carrier sense with randomized resensing delay, was previously analyzed in Packet Radio Temporary Note #51 [F1]. In that analysis, however, the probability of blocking P_b and the probability of conflict P_c were implicitly assumed to be independent; this assumption lead to an erroneous expression for the probability of success P_s and the throughput. For the same protocol of carrier sense presented in [F1] and under the same model assumptions, we give here the correct analysis.

The notation used in [F1] is used here at the exception of the following:

G: normalized rate of the aggregate process of listen time

$$p = \frac{T_p}{T_m}$$

$$d = \frac{T_d}{T_m}$$

$$T_m = 1$$

Analysis

Consider the time axis of the channel on which one observes an alternate sequence of busy periods and idle periods. In this access mode, a busy period consists of a single transmission period. Let B be the point process corresponding to the first packet arrival of each busy period. In Fig. A1, P1 and P2 are two consecutive renewal points of process B. Consider the arrival P1 which occurs at time t_{p1}. The transmitter of packet P1 listens to the carrier for a period equal to d sec., and sensing the channel idle, transmits the packet at time $t_{p1} + d$. Assuming the distance between all pairs of devices is the same (and equal to the largest such distance in order to get lower bounds on throughput) packet P1 is detected present by another terminal if and only if the latter starts sensing the channel beyond time $t_{P1} + d + p$.*

*It is implicitly assumed here that only if the channel is on for the entire d sec. would it be regarded as busy.

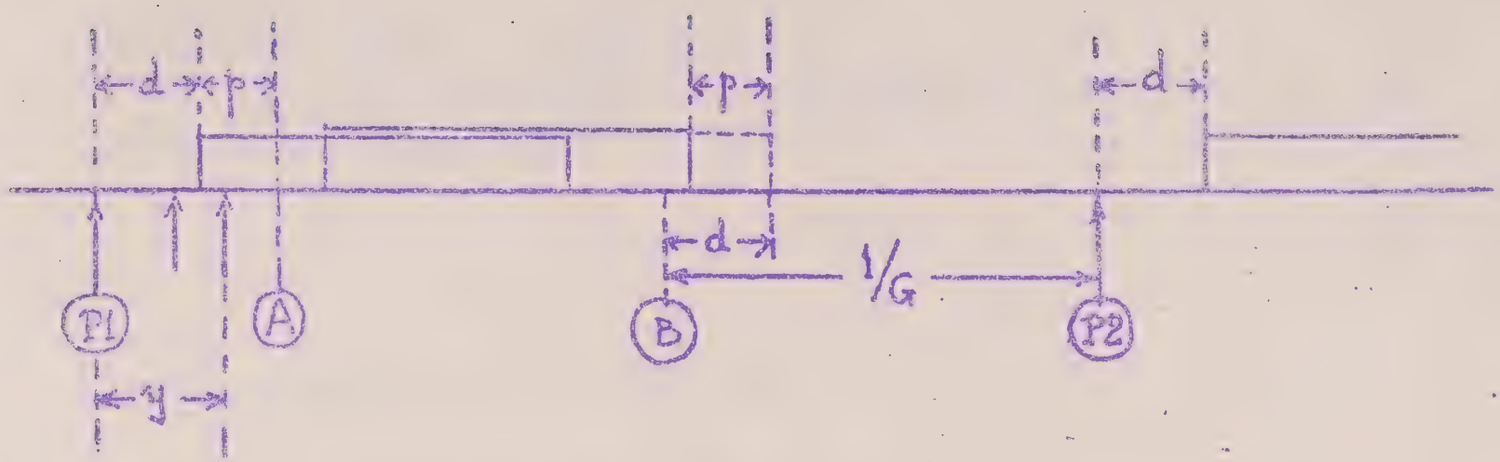


Fig A-1

Therefore, any arrival occuring in the first $d + p$ sec. of a busy period will sense the channel idle and will cause interference. Let $t_{p1} + y$ be the time of arrival of the last packet occuring in the first $d + p$ sec. of a busy period. From [K1], we immediately have:

$$\bar{y} = p + d - \frac{1}{G}\left(1 - e^{-G(p+d)}\right) \qquad a\text{-}1$$

On the other hand, an arrival occuring during the last d sec. of the transmission period (see Fig. A-1) will sense the channel idle and will transmit without causing any interference. Thus, the first arrival occuring beyond point B will constitute the first renewal point after P1; it is denoted in Fig. A-1 by P2.

Consider now an arbitrary cycle defined by two consecutive renewal points (such as P1 and P2). The average cycle duration is

$$\bar{C} = \bar{y} + d + 1 + p - d + \frac{1}{G}$$

$$= \bar{y} + 1 + p + \frac{1}{G} \qquad a\text{-}2$$

A packet is successful if

(i) it is not blocked

and

(ii) it does not conflict with any other packet.

An arbitrary packet is not blocked if:

a. Upon its arrival it finds the channel in the first $d + p$ sec. of the cycle. This occurs with probability

$$(1 - P_b)_1 = \frac{d + p}{\bar{C}} \qquad a-1$$

and the probability of conflict is $P_{c_1} = 1$.

or b. Upon its arrival, it finds the channel in the last $1/G$ sec. of the cycle. This occurs with probability

$$(1 - P_b)_2 = \frac{1/G}{\bar{C}} \qquad a-4$$

and the probability of conflict is $P_{c_2} = 1 - e^{-(p+d)G}$. Since events (a) and (b) are independent, we can write:

$$P_S = \frac{S}{G} = (1 - P_b)_1 (1 - P_{c_1}) + (1 - P_b)_2 (1 - P_{c_2})$$

$$= \frac{e^{-(p+d)G}}{G(1 + d + 2p) + e^{-(p+d)G}} \qquad a-5$$

The cycle analysis can be applied as well leading in a much simpler and more straightforward manner to Eq. a-5.

In the case $d = 0$, $p = a$, Eq. a-5 is written as

$$\frac{S}{G} = \frac{e^{-aG}}{G(1 + 2a) + e^{-aG}} \qquad a-6$$

Eq. a-6 and simulation results (corresponding to $d = 0$) are plotted and compared in Fig. A-2.

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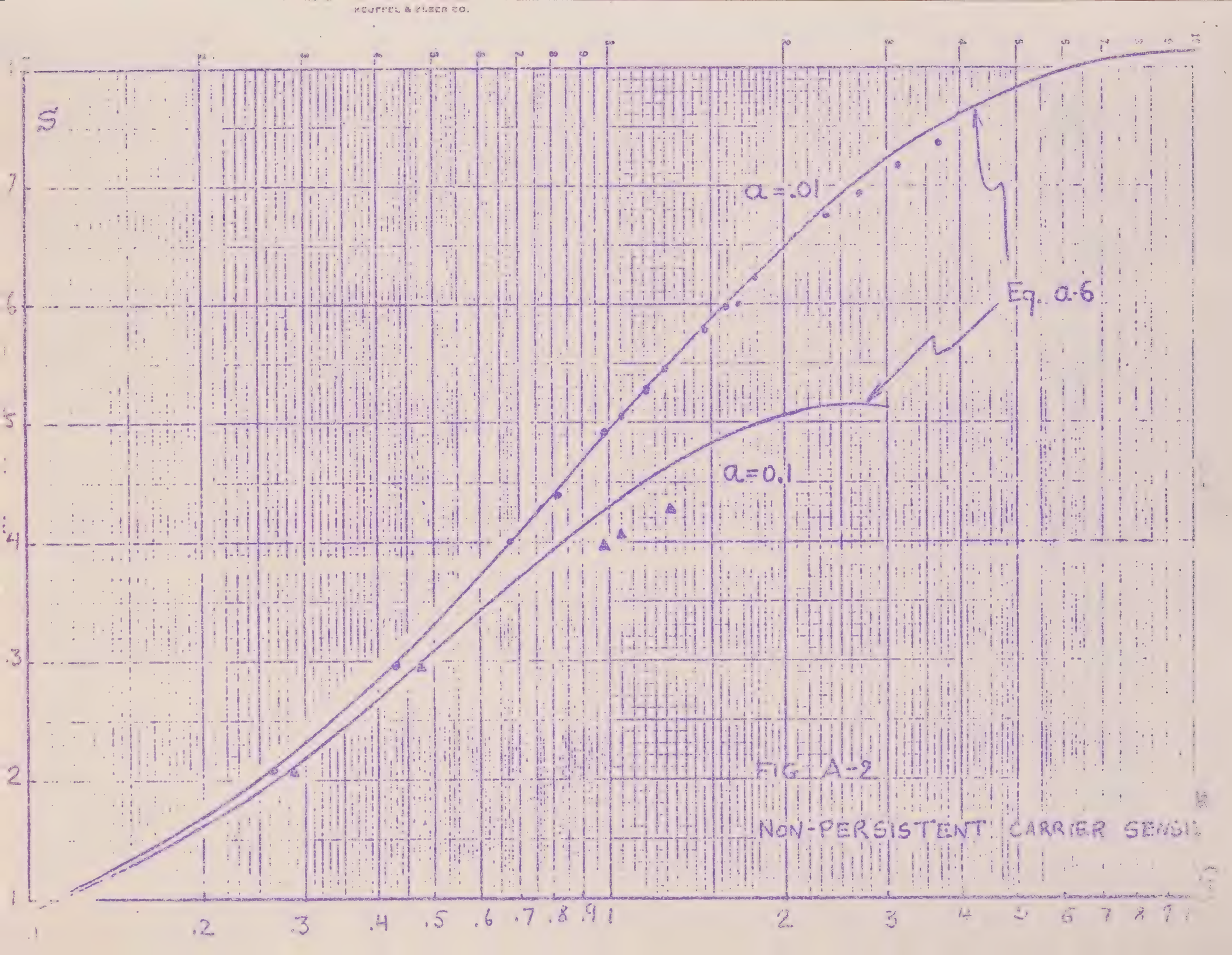


For a = 0.01, we plot in Fig. A-3 delay curves for the three carrier sense access modes: 1-persistent, p-persistent, and non-persistent. These curves are obtained from simulation. Note that the optimum p-persistent carrier sense provides the best performance.

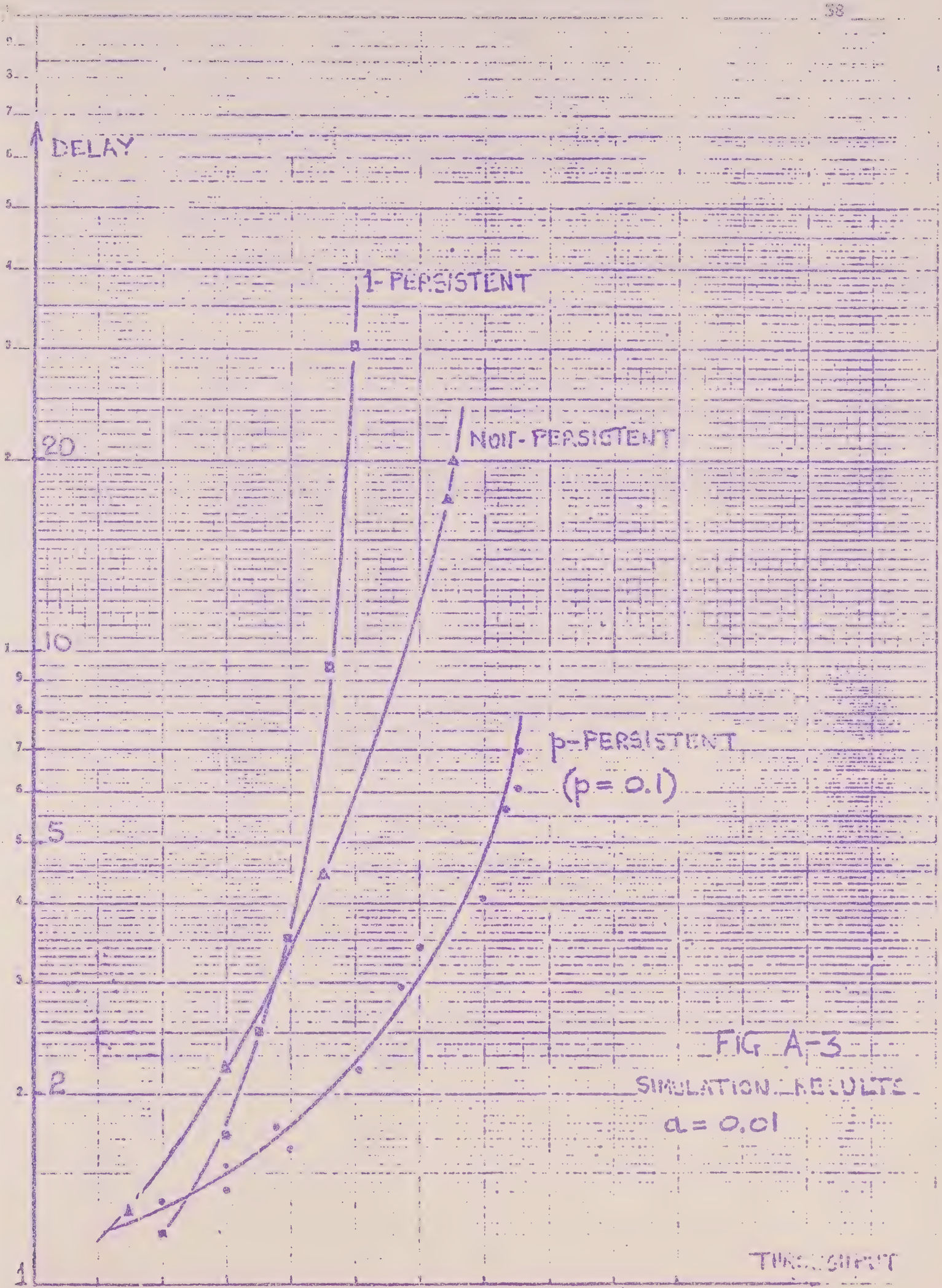


FIG. A-3

SIMULATION RESULTS

a = 0.01

APPENDIX B

PROOFS OF THEOREMS

I. Proof of Theorem 1

Let us consider, for each group i, a time axis called line i, on which we observe an alternate sequence of busy periods and idle periods as defined in [K1].

Since all groups are independent, the sequence observed on line i is completely determined by the traffic rate G_i alone.

A packet from group i is successful if and only if the three following conditions are satisfied:

C1. The packet is successful when group i is considered alone, in the absence of all other groups. This condition occurs with probability [K1]:

$$P_r\{C_i\} = \frac{(G_i + aG_i + e^{-aG_i})(1 + aG_i)e^{-G_i(1+a)}}{G_i(1 + 2a) - (1 - e^{-aG_i}) + (1 + aG_i)e^{-G_i(1+a)}} \qquad b\text{-}1$$

C2. The arrival of the tagged packet does not occur during the transmission time of a packet on any line $j (j \neq i)$. From [K1] we have (see Fig. B-1):

$$\bar{I}_j = \frac{1}{G_j} + a$$

$$\bar{B}_j = \frac{1 + a + \bar{y}_j}{q_{oj}} - a$$

$$\bar{y}_j = a - \frac{1}{G}(1 - e^{-aG_j})$$

$$q_{oj} = e^{-G_j(1+a)}(1 + aG_j)$$

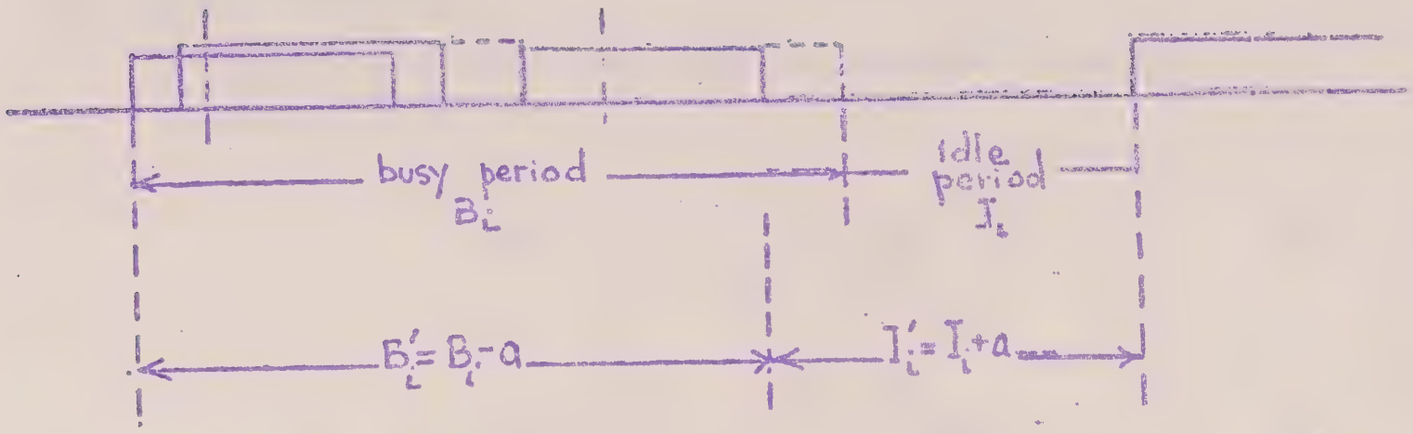


FIG. B-1

Therefore

$$P_r\{C_2\} = \prod_{\substack{j=1 \\ j \neq i}}^{N} \frac{(1 + aG_j)^2 e^{-G_j(1+a)}}{G_j(1 + 2a) - (1 - e^{-aG_j}) + (1 + aG_j)e^{-G_j(1+a)}} \qquad \text{b-2}$$

C3. Given C_2, there are no arrivals from group j, $j \neq i$, during the entire transmission time of the tagged packet.

$$P_r\{C_3\} = e^{-\sum_{\substack{j=1 \\ j \neq i}}^{N} G_j} \qquad \text{b-3}$$

Finally

$$P_{S_i} = P_r\{C_1\} \cdot P_r\{C_2\} \cdot P_r\{C_3\}$$

which lead to Eq. 1.

II. Proof of Theorem 2:

The proof is the same as for Theorem 1, in which, by referring to Appendix A we have

$$P_r\{C_1\} = \frac{e^{-aG_i}}{G_i(1 + 2a) + e^{-aG_i}} \qquad \text{b-4}$$

$$P_r\{C_2\} = \prod_{j\neq i} \frac{(1 + aG_j)}{G_j(1 + 2a) + e^{-aG_j}} \qquad \text{b-5}$$

and

$$P_r\{C_3\} = e^{-\sum_{j\neq i} G_j} \qquad \text{b-6}$$

III. Proof of Theorem 3:

P_r\{an arbitrary packet from group i starts a busy period\} =

P_r\{the packet arrives during the idle period of line i and is not blocked by groups j, $j\in h(i)$, $j\neq i$\}

which under assumption A'2 can be written as:

= P_r\{the packet arrives during the idle period of line i\} X

$$\prod_{\substack{j\in h(i)\\ j\neq i}} P_r\{\text{the packet is not blocked by group } j\}$$

The average time lapse between two consecutive renewal points of process P_i is $1/H_i$. Therefore (see Fig. B-2)

P_r\{the packet arrives during the idle period of line i\} =

$$\frac{\frac{1}{H_i} - (1 + \bar{y}_i + a)}{\frac{1}{H_i}} = 1 - H_i(1 + \bar{y}_i + a)$$

and (see Fig. B-3).

$$P_r\{\text{the packet is not blocked by group } j\} =$$

$$\frac{\frac{1}{H_j} - (1 + \bar{y}_j)}{\frac{1}{H_j}} = 1 - H_j(1 + \bar{y}_j)$$

Finally

$$H_i = G_i[1 - H_i(1 + \bar{y}_i + a)] \cdot \prod_{\substack{j\in ch(i) \\ j\neq i}} [1 - H_j(1 + \bar{y}_j)] \qquad b\text{-}7$$

IV. Proof of Theorem 4:

A busy period on line i is successful if and only if the following conditions are satisfied:

C_1': The starting point of the tagged busy period does not occur during the first a sec. of a busy period on line j, $j\in ch(i)$, $j\neq i$.

$$P_r\{C_1'\} = \prod_{\substack{j\in ch(i) \\ j\neq i}} \frac{1 - H_j(1 + \bar{y}_j + a)}{1 - H_j(1 + \bar{y}_j)}$$

C_2': Among all groups $j\in ch(i)$, there are no arrivals during the first a sec. of the tagged busy period.

$$P_r\{C_2'\} = \prod_{j\in ch(i)} e^{-aG_j} = e^{-a\sum_{j\in ch(i)} G_j}$$

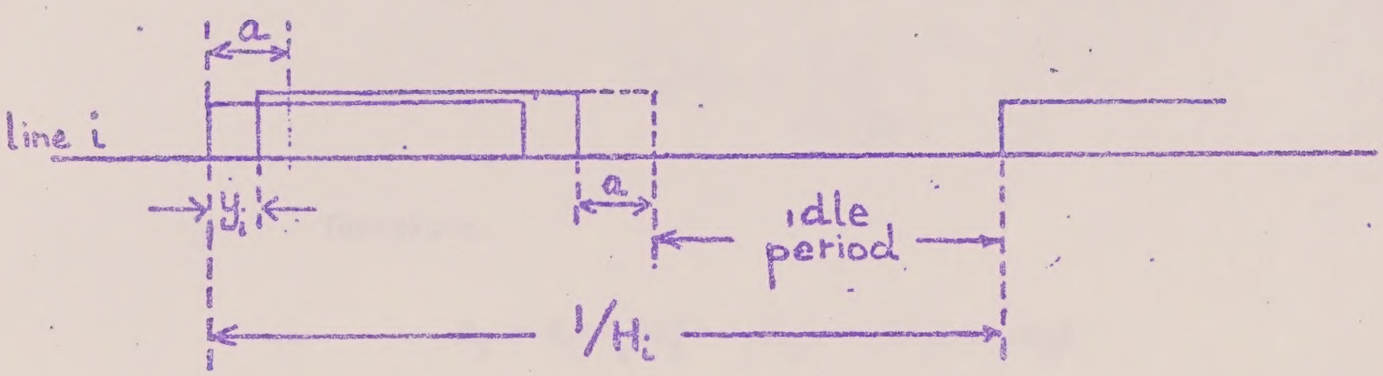
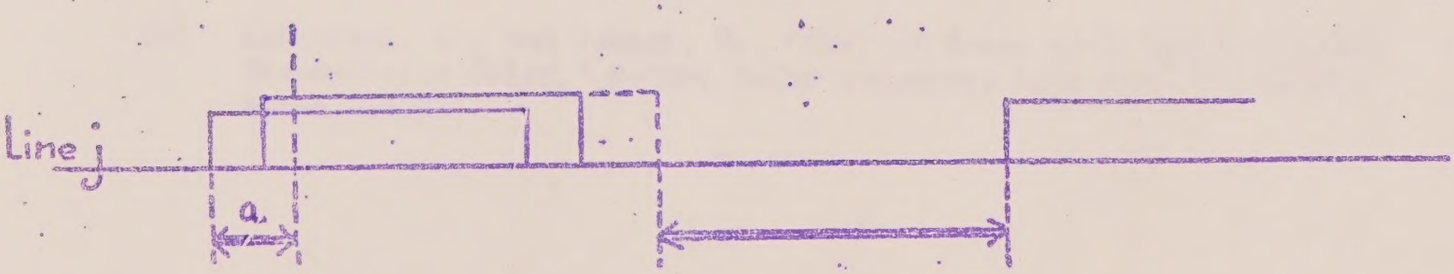
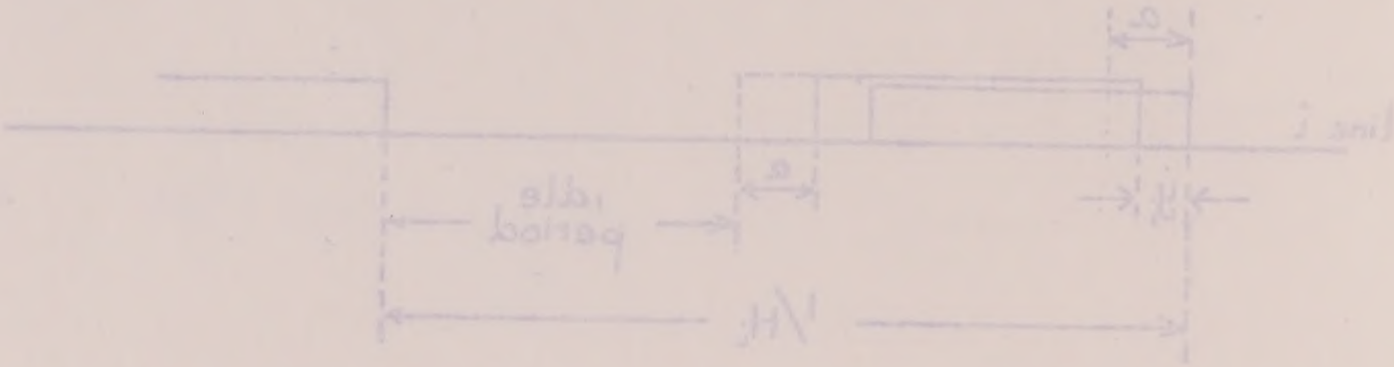


FIG. B-2



⟷ : period when packet from group i is not blocked by group j , $j \in h(i)$

FIG. B-3



C_3': The arrival of the tagged busy period does not occur during the transmission time of a packet on any line k, $k \notin h(i)$.

$$P_r\{C_3'\} = \prod_{k \notin h(i)} [1 - H_k(1 + \bar{y}_k)]$$

C_4': Given C_1', C_2', and C_3' there are no arrivals from group k, $k \notin h(i)$, during the entire transmission time of the tagged packet.

$$P_r\{C_4'\} = \prod_{k \notin h(i)} e^{-G_k}$$

Therefore:

$$S_i = H_i \cdot P_r\{C_1'\} \cdot P_r\{C_2'\} \cdot P_r\{C_3'\} \cdot P_r\{C_4'\}$$

Substituting H_i from Eq. b-6, we get Eq. 4.

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[1] Fralick, S., Leung, J., "Preliminary Comparison of Different Access Modes,"Packet Radio Trmporary Note #51, NIC 16742, June 1, 1973.

[2] Kleinrock, L., and Tobagi, F., "Throughput in Carrier Sense (Auto Slot) Packet Radio Systems," Packet Radio Temporary Note #24, NIC T 14212, Feb. 12, 1973.

[3] Kleinrock, L., and Tobagi, F., "Carrier Sense with Initial Random Transmission Delay," Packet Radio Temporary Note #37, NIC 15334, March, 1973.